Proposal for Funding of the Project:
The Multi-Fractal Model of Asset Returns: Multivariate Extensions, Estimation, and Applications for Risk Management

Antrag auf Gewährung einer Sachbeihilfe eingereicht von:

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1 Proposal for Funding of the Project:

The Multi-Fractal Model of Asset Returns: Multivariate Extensions, Estimation, and Applications for Risk Management. This a new project that has not received financial support by the DFG or any other research funding agency before.

1.1 Applicant

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1.2 Topics

Fractality, long memory, Markov-switching, realized volatility and Student-t innovations in asset returns. GMM estimation, Maximum Likelihood, Simulated Maximum Likelihood and non-parametric estimation techiques for multifractal models of asset returns. Applications to forecasting and risk management for different asset types and cross-sections.

1.3 Research Areas

Financial Econometrics, Risk Management, Econophysics

1.4 Anticipated Duration

A total of 3 years. The funding by the DFG would be requested for the total anticipated duration of the project.

1.5 Proposal Period

36 Months
Desired commencement of the project: 01.05.08
Funded persons: Leonardo Morales-Arias, M.Sc. for the first period of about 12 months, who will be followed by Payam Norouzzadeh, M.Sc. in months 13 through 36.

1.6 Summary of the Project

Multifractal processes have been originally developed in physics for modeling turbulent fluids and related phenomena. They have recently also attracted attention in empirical finance because of their ability to replicate the major stylized facts of asset returns: varying degrees of long-term correlation of different measures of volatility, and fat tails in the unconditional distribution of price changes. While the multifractal apparatus developed in statistical physics is mainly concerned with combinatorial operations on measures, analogous causal multifractal models have been designed for financial applications. Although this literature is still at a very early stage, it has already developed a range of statistical techniques for proper estimation of multifractal models and has demonstrated successful applications in forecasting of volatility.

The present project builds upon the earlier work on inference methods for multifractal models and their practical applications by our group. One major task will be the development of appropriate statistical methodology for *multivariate* multifractal models. We will investigate the behavior of various extensions and explore the use of multifractal models for risk management and portfolio management. Further research includes the analysis of the role of innovations vis-à-vis the intrinsic volatility dynamics as well as the adaptation of the multifractal model to measures of realized volatility. Given the evidence on multi-scaling of many physical time series, we also expect that the methodological innovations in financial applications will generate spill over effects to the application of multifractal models in the natural science (for example, improved forecasts of precipitation).

2 State of the Literature and Own Work on the Subject

2.1 State of the Literature

While so-called uni-fractal of self-similar processes (like fractional Brownian motion) are widely used in financial economics, more general multi-fractal processes have only been considered very recently as candidate data-generating mechanisms for financial prices. However, the typical signature of *multi-fractality*, namely variations in the *scaling behavior* of different moments of the data, is a well-established feature of price changes in both stock and foreign exchange markets. In the economics literature (which did not use the concept of scaling or fractality until recently) the equivalent feature of variations in temporal dependence of various powers of the raw data has been reported first by Ding, Granger and Engle (1993) and has been confirmed by a number of later studies (e.g. Lux, 1996; Mills, 1997; Lobato and Savin, 1998). Almost at the same time, a number of papers authored by physicists (Schmitt, Schertzer and Lovejoy, 1999; Vandewalle and Ausloos, 1998a,b; Vassilicos, Demos and Tata, 1993) using a different set of analytical tools arrived at essentially the same conclusions concerning the multi-fractal nature of various financial records. These findings have also stimulated comparisons between the statistical behavior of data from both financial markets and experimental data of turbulent flows (Vassilicos, 1995; Mantegna and Stanley, 1996; Voit, 2001).

The basic principle for construction of multifractal models in statistical physics is a cascading process of iterative splitting of initially uniform probability mass into more and more heterogeneous subsets. Starting with a uniform distribution over a certain interval, one splits this interval into two subintervals that receive fractions, say p_1 and $1 - p_1$, of the overall mass. In the next step, the same procedure is repeated for the newly created subsets so that one ends up with four intervals with probability mass p_1^2, $p_1(1-p_1)$ and $(1-p_1)^2$, respectively.[1] In principle, this process can be repeated *ad infinitum*. One thus obtains a hierarchical structure of components, where smaller ones (small whirls is the application to turbulent flows) emanate from the higher levels of the hierarchy via this probabilistic split of energy. By its very construction, a combinatorial multifractal along the above lines exhibits different degrees of scaling or long-term dependence for different powers of the resulting measure. This makes adaptation of the multifractal apparatus a promising avenue of reseach in empirical finance.

Interestingly, empirical research in finance also provides us with more direct evidence in favor of the hierarchical structure implied by these cascade models: namely,

[1]The two intervals in the center both have measure $p_1(1-p_1)$.

it has been found that volatility on fine scales (tick-by-tick data) can be explained to a larger extent by coarse-grained volatility than *vice versa* (Müller et.al., 1997). This feature would be expected in a multifractal generating process, but not for traditional models of asset returns. Although the analogy between both turbulence and financial prices does not hold in all details, some authors have proposed to formalise the financial multi-fractality using cascade models developed for velocity differences in turbulent flows (e.g. Ghasghaie et al., 1996). Recently, Calvet, Fisher and Mandelbrot (1997) proceeded one step further by proposing a compound stochastic process as a data generating mechanism for financial prices in which a multi-fractal cascade plays the role of a time transformation or time-varying scale function of the variance of the incremental process.[2] In their model[3] an incremental Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, this multifractal component is of combinatorial rather than causal nature (it is actually identical to the model proposed for turbulent flows by Mandelbrot, 1974).

Unfortunately, the original MMAR suffers from non-stationarity since the combinatorial construction of the multifractal measure is restricted to a (predefined) bounded interval (in space or time). While Calvet and Fisher (2002) develop estimators and diagnostic tests for this particular model, its non-causal nature makes it intrinsically difficult to apply to financial data. As a consequence, although the potential of this new approach for generating multi-scaling in returns is not shared by traditional models in finance, rigorous comparison of its performance to, for example, more traditional GARCH processes (Bollerslev, Chou and Kroner, 1992), is hampered by the lack of statistical theory for the parameter estimates and statistical tools for comparison of alternative models.

These severe limitations have been overcome by the development of an iterative, causal analogue of the combinatorial MMAR, cf. Breymann et al. (2000) and Calvet and Fisher (2001). Calvet and Fisher define a continuous-time multifractal model with random times for the changes of its volatility components and demonstrate weak convergence of the discretized version of this process to its continuous-time limit. This approach preserves the hierarchy of volatility components of the original MMAR but dispenses with its restriction to a bounded interval. In the discrete-time version of this model, the volatility dynamics is driven by a Markov-switching process with a large number of states. One important advantage of the multifractal model is that this large state space does not come along with a similarly large number of parameters. Quite in contrast to standard Markov-switching models, the hierarchical structure leads to

[2] Cf. also Mandelbrot (1999) and Mandelbrot and Hudson (2004) for a non-technical introduction to the subject and the relationship between the MMAR and multifractal models in turbulence.

[3] denoted the *Multifractal Model of Asset Returns* (MMAR)

a transition matrix whose probabilities are determined by only a few parameters, or might even be defined in a way to be parameter-free.

To concretize ideas, we will reproduce here the structure of the Markov-Switching Multifractal Model (MSM). Returns are modeled as

$$r_t = \sigma_t u_t \tag{1}$$

with innovations u_t taken from a stationary distribution (e.g. the standard Normal distribution). Instantaneous volatility σ_t is determined by the product of k volatility components or multipliers $M_t^{(1)}, M_t^{(2)} \ldots$ and a scale factor σ:

$$\sigma_t^2 = \sigma^2 \prod_{i=1}^{k} M_t^{(i)}. \tag{2}$$

Following the basic hierarchical principle of the multifractal approach, each volatility component will be renewed at time t with a probability γ_i depending on its rank within the hierarchy of multipliers and remains unchanged with probability $1 - \gamma_i$. The convergence property demonstrated by Calvet and Fisher (2001) requires to formalize transition probabilities according to:

$$\gamma_i = 1 - (1 - \gamma_k)^{(b^{i-k})} \tag{3}$$

with γ_k and b parameters to be estimated. However, a large part of the literature uses simpler specification of hierarchical probabilities (used in, for example, Liu and Lux, 2006 and Lux, 2007) such as:

$$\gamma_i = 2^{-(k-i)} \tag{4}$$

which also allows for components of very different life-times. While the parameter-free version of eq. (4) greatly facilitates estimation, it is also germaine to the structure of renewal of multipliers in the physical applications of multifractals and the previous MMAR. The MSM model is fully specified once we have determined the number k of volatility components and their distribution. In the small body of available literature, the multipliers have been assumed to follow either a Binomial or a Lognormal distribution. Since one would normalize the distribution so that $E[M_t^{(i)}] = 1$, only one parameter has to be estimated for the distribution of volatility components in these cases. Taking into account the scale parameter, σ, we end up with a very parsimonious family of stochastic processes that is parameterized by only two parameters although the number of states could be arbitrary large (for large k).

Calvet and Fisher (2004) develop a maximum likelihood algorithm (MLE) for estimation of the parameters of this MSM model. Since MLE requires an evaluation of the transition matrix, it works only for discrete distributions of the multipliers and is not applicable for, e.g., the alternative proposal of a Lognormal distribution. It also encounters bounds of computational feasibility for specifications with more than about 10 volatility components. Despite these limitations, the estimated Binomial MSM specifications in Calvet and Fisher (2004) did provide gains in forecasting accuracy of future volatility over standard GARCH and fractional GARCH models (FIGARCH) over medium-term and long horizons. Calvet, Fisher and Thompson (2006) add two additional innovations to this literature: first, they propose a bi-variate generalization of the Markov-Switching Multifractal Model (which is different from the one by Liu and Lux which had been developed independently). Second, they develop a simulated ML or particle filter approach for estimation of the parameters that is somewhat more broadly applicable than the MLE algorithm of their previous paper.

2.2 Own Work on the Subject

The applicant and his group have been working on various aspects of this new class of stochastic volatility models over the last couple of years. Lux (2001) provides parameter estimates for two specifications of the early combinatorial MMAR and demonstrates that simulations of the estimated models yield an unconditional distribution that is much closer to the empirical distribution than returns from standard GARCH-type models. Another paper (Lux, 2004) demonstrates the unreliability of the so-called ‘scaling’ estimator adopted from statistical physics by the early literature. At least for the subordinate multifractal processes of Mandelbrot et al. (1997) and Calvet and Fisher (2002), this popular estimator is shown to give extremely volatile parameter estimates and tests based on this estimator are unable to reject the null hypothesis of multi-fractal behavior for uni-fractal processes. Lux (2007) takes up the question of estimating the ‘second generation’ Markov-Switching Multifractal model. This paper introduces a Generalized Method of Moments (GMM) estimator that is universally applicable to all possible specifications of MSM processes. In particular, it can be applied in all those cases where ML is not applicable or computationally unfeasible. In all cases, it is advantageous computationally as it requires CPU time of (at least) one or two orders of magnitude less than that of ML or SML estimation algorithms. In order to forecast volatility, these estimates are combined with best linear forecasts (Brockwell and Dahlhaus, 2004) on the base of analytical autocovariances of the process. Numerical experiments show that despite the loss of efficiency implied by using GMM rather than ML, the accuracy of forecasting is almost the same. The empirical application shows the advantage of new specifications of MSM models that can be handled via GMM but not by ML estimation. In particular, allowing for a larger state space (k higher than 10) often allows for further improvements in forecasting power against the specifications explored by Calvet and Fisher (2004).

Lux and Kaizoji (2007) perform a more comprehensive forecasting exercise with the MF asset pricing model for the Tokyo Stock Market using GMM estimation together with best linear forecasts. It is found that the GMM-MF model can outperform both short and long memory models in terms of forecastability. More precisely, it is shown that long memory models such as FIGARCH, ARFIMA and the GMM-MF outperform short memory models such as GARCH and ARMA models. Morever, when comparing the long memory models against each other, it is found that the GMM-MF model often improves upon the forecasts of both FIGARCH and ARFIMA models. Lux and Kaizoji also show that an appropriately modified version of the MSM model can also be used as a model for trading volume and again performs better in forecasting this quantity than more standard time series models like ARMA and ARFIMA.

Some other aspects of multifractal models and their applications have been successfully dealt with in recent Ph.D. theses under the supervision of the applicant. Lee (2007) considers multinomial extensions of the Binomial version of MSM which improves their in-sample and out-of-sample fit in comparison to the baseline Binomial specification. Liu, di Matteo and Lux (2007) have explored the behavior of estimated MF models under various statistical tests. Liu and Lux (2006) extend the MF model to the bivariate case and estimate their bivariate multifractal model (BMF) via GMM and Maximum Likelihood. The BMF asset pricing model is assesed empirically by computing the Value-at-Risk of portfolios employing stock, bond and foreign exchange rate indices.

In subsequent work, Liu (2007), the multi-variate approach of Liu and Lux (2006) is contrasted with the alternative multi-variate model published by Calvet, Fisher and Thompson (2006). Both approaches are characterized by somewhat different mechanisms for the correlation of volatility between assets: Calvet et al. assume that the distributions of volatility components might have different parameters for both assets, but that they are correlated over all hierarchical levels. Liu and Lux (2006), in contrast, suppose that the parameters are the same, but that volatility correlation is restricted to a subset of components $k_1 < k$. In Liu's comparative analysis of both specifications, the Liu and Lux (2006) version appeared to be somewhat better able to capture the empirical distribution of Value-at-Risk in various bi-variate test portfolios.

3 Proposed Research Programme for the Project

3.1 Objectives

As previously indicated one of the main aims of this project is to extend the bivariate multifractal model proposed by Liu and Lux (2006) for forecasting and risk management. For this purpose we will start by resorting to the best linear forecasts computed via the Durbin-Levinson algorithm given the highly non-linear structure of the BMF model along the lines of Lux (2007) for the uni-variate case. A first important step will be the generalization of the so far relatively preliminary state of development of these forecasts. Best linear forecasts are computed *separately* for two assets of a simple portfolio in Liu and Lux (2006). In order to fully exploit the rich structure of dependencies offered by MF models, this approach needs to be generalized by the development of truely multi-variate forecasts taking into account the structure of autocorrelations between assets. These more accurate forecasts of portfolio risk would then have to be generalized for large portfolios with an arbitrary number of assets.

For these MMF portfolio forecasts, we will design a comprehensive framework for forecasting VaR which can be summarized in four main steps. First, we will apply rolling windows and recursive procedures to investigate the optimal scheme when forecasting with the BMF model. Studies by Granger (1989) and Aiolfi and Timmermann (2006) show that it is often preferable to combine alternative forecasts of different forecasting schemes or nested models in a linear fashion and thereby obtain a new predictor. Thus, we will also consider whether optimal forecast combinations (e.g. MMF models with normal and t-distributed innovations) can improve forecastability in comparison singular schemes.

Second, a rigorous set of forecast evaluation techniques will be employed. In particular, we will employ (i) a CUSUM type test for parameter stability, (i) a test on average predictive accuracy such as the one proposed by Diebold and Mariano (1995), (ii) a test on forecast encompassing as proposed by Harvey et.al (1998), (iii) a test of out-of-sample distributional features of returns as proposed by Hendriksson and Merton (1981) and (iv) a test for serial correlation on forecasting errors. Steps 1 to 2 will be carried out for different frequencies of the data (tic by tic, daily, monthly, quaterly).

Third, we will employ state-of-the-art tests for Value-at-Risk that consider both the unconditional coverage hypothesis (the probability of an *ex-post* loss exceeding VaR forecasts must be exactly equal to the the coverage rate) and the independence hypothesis (VaR violations observed at two different dates for the same coverage rate must be distributed independently). Tests that consider these two hypothesis for VaR forecasts can be subdivided into two main groups, namely, the event probability forecast approach (Christoffersen (1998), Engle and Manganelli (2004), Berkovitz et al. (2005)) which deals with conditional efficiency for a given nominal coverage rate and the density forecast approach (Crnkovic and Drachman (1997), Diebold et al. (1998) and Berkowitz (2001)) which consider conditional efficiency for any coverage rate.

Finally, our last step will be to compare the VaR forecasts of the GMM-BMF model against state-of-the-art multivariate volatility models along the lines of Lux and Kaizoji (2007) for the univariate MF model. Since we will be dealing with a bivariate model we will resort to bivariate volatility competitors for both the short memory case (BEKK of Engle and Kroner (1995) and DCC of Engle (2002)) and the long memory case (BFIGARCH of Brunetti and Gilbert (1998) and RV-VAR of Andersen et.al. (2003)).

Similar experiments will be conducted for the alternative specifications of multivariate MF models proposed by Calvet, Fisher and Thompson (2006). A comprehensive sensitivity analysis will explore strengths and weaknesses of different formalizations vis-à-vis each other as well as the interaction between the statistical method used for parameter estimation and forecasting (ML, SML, GMM) and the accuracy of the resulting forecasts.

The implementation of the subsequent steps of our research will be influenced by our findings in this first part. Further extensions of the scope of our research program will include:

- the exploration of different distributions of the innovations. Lee (2007) has already made an attempt at extending the GMM estimation algorithm to innovations drawn from a Student t distribution. While a more flexible distributional form, of course, should increase in-sample fit, it is not clear wether it also leads to improvements in out-of-sample forecasts. This issue will be investigated along the lines of our above outline of the research to be conducted on the baseline MF model. In order to be able to assess the performance of the MF-Student to forecast value-at-risk, a multivariate extension of the MF-Student model will be developed. Even more distributional flexibility could be gained by resorting to more general families of distributions. An interesting recent example is Abraham et al. (2007) who apply Gamma distributed marginals within the context of a stochastic volatility model. In cases where there is pronounced skewness or asymmetry between the right wing and left wing of the distribution, empirical applications could benefit from such added flexibility. Given our previous work, incorporation of any type of incremental distribution should be feasible in principle so that we can also analyze such alternatives.

- application of the MF model to measures of *realized volatility*. A large body of recent literature has replaced simpler measures of volatility (like squared or absolute returns or sample variances over rolling windows) by the sum of intraday squared returns. Anderson and Bollerslev (1998) show that this measure is a better proxy for the unobservable daily volatility dynamics than the simpler alternatives, at least if sampling of intra-daily observations is sufficiently frequent. Phenomenological studies of realized volatility show that it behaves like a long-memory process (Anderson et al., 2001). A large number of recent papers has investigated the forecasting accuracy of statistical models using realized volatility as information (e.g. Andersen et al., 2003; Pong et al., 2004). Due to their principle of construction, time series of realized volatility can not be modelled by GARCH-type models, so that researchers in this area had to resort to alternative time series models like ARMA and ARFIMA. The multifractal model seems to be another natural candidate as it can, in principal, be implemented on the base of intra-daily data. As an advantage against AR(FI)MA models, volatility is guaranteed to be non-negative by construction in the MF model. Extending our previous work on MF models to measures of realized volatility requires certain conceptual modifications, however. While the previous applications to daily data assumed the lowest volatility component within the hierarchy of multipliers to undergo changes with daily frequency, the new application requires sensible assumptions on 'submerged' components at even higher frequency (i.e. intra-daily). While it is not clear a *priori* (and will be subject of our research) how exactly an appropriately modified framework would look like, the formal apparatus developed previously for estimation and empirical applications of MF models would certainly be general enough to accommodate this important extension.

- comparison with alternative approaches to multifractal modelling and non-parametric reconstruction of the hierarchical components of systems with multiple scales. An entirely different way to 'translate' the multifractal apparatus into causal iterative processes has been proposed recently by Gutierrez and Rodriguez (2000). These authors use estimated multiscaling characteristics in order to select the parameters of interated function systems (IFS) sharing these features. Wang et al. (2007) apply this method to develop forecasts of precipitation by pattern recognition techniques using empirical data and simulated records from the estimated IFSs. Obviously, this is an approach quite different from our more direct forecasting algorithm. It would, therefore be worthwhile to explore the applicability of the algorithm to financial data. At the same time, reliance on the cumbersome 'scaling' estimator in Gutierrez and Rodriguez (2000) raises the question whether this identification of IFSs could be improved by the statistical methodology developed for the Markov-switching multifractal model. Another recent innovation is the development of non-parametric methods to reconstruct the multi-scale dynamics of hierarchical processes (Renner, Peinke and Friedrich, 2001a,b, Nawroth and Peinke, 2006). Their approach consists in determining the shift and diffusion components of the transient conditional probability density function (pdf) of a multiscale process. The basic invention here is the introduction of a generalized Fokker-Planck equation for the time development of the scale dependent pdf. With polynomial approximation of the coefficient of the Fokker-Planck equation, parameters can be estimated empirically and it becomes possible to compute forecasts by iteration of the joint multiscale transient probability density. Again, comparison of the performance of this method with our parametric approach would be of interest to shed light on the advantages and disadvantages of various avenues for multiscale modelling of financial data.It would also be interesting to explore how these non-parametric techniques behave in the presence of multifractal data. In addition, the multi-scale Fokker-Planck technique might provide inspiration for improvements of our estimation methods for the parametric MSM model. Besides this direct connection, a more indirect link exists insofar as similar avenues have been pursued by our research group in the estimation of behavioural models with interacting agents (Lux, 2007).

3.2 Schedule for the project

During the first year, we will mainly focus on the development of multivariate extensions of our previous univariate and bivariate MSM models. We will generalize the GMM estimation technique and Durbin-Levinson forecasting approach for multivariate multifractal models. As it is standard in financial econometrics, the performance of these algorithms will be explored in comprehensive Monte Carlo simulations. These experiments will provide a guideline for the particular implementation we will use for the empirical data. For example, Monte Carlo simulations will provide evidence on what sets of moments and how many moments one shall use for the maximum efficiency of GMM estimation. The subsequent empirical application will include a detailed comparison with the performance of multivariate MF models vis-à-vis more traditional GARCH structures as outlined above. We also plan to start during the first year with the exploration of the influence of different distributions of the innovation terms. This part of our research will also be examined via intensive simulation experiments and empirical applications under the guidance of Monte Carlo experiments. We expect this line of research to continue during the second year.

In the second year we will start with our venture into more fundamentally new and unknown territory. We will attempt to clarify how to apply the multifractal apparatus to realized volatility. Together with the development of the conceptual framework, we will establish a data archive of realized volatility from intra-daily data of the New York Stock Exchange and Frankfurt Stock Exchange and perform exploratory analysis with standard time series models against unifractal ones (e.g. ARFIMA). One important question to attack with intra-daily data and realized volatility is the optimal level of aggregation, i.e. should one focus on realized volatility for daily data or would it be preferable to model and forecast volatility over higher frequencies. Related to this is the determination of the highest frequency up to which multi-scaling properties can be observed and exploited. As it is well known, for very high frequency (minute to minute or tick-by-tick) the empirical characteristics of asset returns are dominated by microstructure features (e.g., there is a deviation from pure martingale behavior for time horizons of up to 15 minutes). It is therefore likely that multiscaling breaks down beyond a certain frequency. Once an appropriate formalization of the MF apparatus for realized volatility has been developed, our practical interest will be again in the performance of the MF model in forecasting and risk management against alternative models and the performance of the MF model using realized volatility against its application to simpler volatility measures (i.e. squared returns).

During the second phase of the project, we also attempt to compare our MF models to multifractal volatility on the base of iterated functions systems and non-parametric techniques. We will conduct controlled experiments to find out whether these techniques have similar performance in comparison to our MSM approach or not in modelling and forecasting data. We hope to be able to perform experiments not only with financial returns, but also with other data sets (e.g., precipitation) with supposed multifractal features. Such a comparison would presumably shed light on the shared characteristics and differences between various data with multiple scaling laws. If various multiscaling methods extract different bits of information, forecasts could again be improved by optimal combination of models.

4 Required Funding

4.1 Cost of the Personnel

Besides the research input by the applicant and occasional input by other members of our team (sec. 5.1), a 0.75 position of an appropriately trained scientist with salary according to the 13 TV-L scheme is required for successful completion of this project. A 3/4 position is required because of the excellent job opportunities outside academia for specialists in financial economics and econometrics who could hardly be attracted with a 50% position. The position will be filled by Leonardo Morales-Arias (M.Sc. in Economics, University of Cape Town) in order to carry out the first stage of research of the specified project. Leonardo Morales-Arias is currently a doctoral candidate at the University of Kiel. For the last two years he has been working on the frontier of empirical finance issues such as in-sample analysis and out-of-sample forecasting with dynamic heterogenous panels, estimation and analysis of nonstationary panels, non-linear error correction in panels, application of realized volatility models to empirical finance, identification through heteroskedasticity with panel data, and univariate/multivariate volatility models. He has been working closely to Prof. Dr. Helmut Herwartz (Chair of Econometrics at the University of Kiel) for the last two years and has already authored two working papers. After this first stage of the project (first year), the responsibility will be handed over to a new researcher from the next cohort of Ph. D. students. During the second part of the project, the designated holder of the research position is Payam Norouzzadeh (M.Sc. in Physics, Sharif University of Technology) who entered our Ph.D. program in the fall of 2007. Payam Norouzzadeh has already authored a number of journal articles on the multifractal apparatus from statistical physics. He has also already gained experience on handling large financial data sets in his previous academic education and as a practitioner in the financial industry. With his switch to economics, he provides a very suitable set of qualifications for this interdisciplinary project. This structure of succession of researchers-in-charge is motivated by the the introduction of a formal system of Ph.D. education with a course system during the first year. Due to the role of the Ph. D. courses in providing systematic training in advanced subjects and guidance towards ongoing research projects within the department, we decided deliberately to deviate from previous traditions and have designed the project for two Ph. D. students in their advanced stage. Both researchers will be given the opportunity to complete their Ph.D theses in addition to the work within the project.

The data-intensive and computation-intensive nature of our project also requires an additional position for a student assistant to carry out work on the maintenance of our data-bases, and preparation of data in the sense of transformation of formats, removal of seasonalities, outliers, etc. Typical working time at the University of Kiel is for 30 hours/month. The occupation of student assistants will also serve to introduce them into empirical work on high frequency financial data and will be combined with their training in seminars and diploma theses on relevant topics.

4.2 Travel Expenses

3000 Euro/year. This stipulated amount would be used for fees, airplane tickets and accommodation expenses at relevant conferences as well as for annual meetings between our group and the group of Professor Peinke at Oldenburg. Conferences at which we would typically present our research are: the Annual Meeting of the Society for Computational Economics, the annual conference "Forecasting of Financial Markets" as well as various more specialized finance conferences and interdisciplinary meetings in "econophysics".

4.3 Publication Costs

750 Euro/year.

4.4 Miscellaneous Costs

1500 Euro /year. This stipulated amount would be used for part of the additional costs that arise during the project period including mainly consumables (such as printer cartridges) and another license for the computer language GAUSS needed for this project (costs will be split between the project and the main budget of the chair).

5 Prerequisites for the Intended Project

5.1 Composition of the Team (as of 1 Jan 08)

- Prof. Dr. Thomas Lux
- Junior-Prof. Dr. Simone Alfarano
- Dr. Reiner Franke (Research Assistant)
- Mishael Milakovic, PhD (Post-Doctoral researcher within a project funded by the Volkswagen Foundation)
- Leonardo Morales-Arias, MSc. (Research Assistant within a project funded by the European Commission ending in April 08)
- Dipl.-Volkswirt Matthias Radant (Research Assistant)

- Dipl.-Volkswirtin Xiaokang Wang (Researcher within a project funded by the European Commission)

While various other research projects are pursued within our team, a number of interfaces with the present proposal exist: Practically all current research projects involve aspects of behavioral or stochastic modelling of financial data. Therefore, a broad range of competences exists within the group concerning data, models and pertinent statistical/econometric techniques and programming tools.

5.2 Cooperation with Local Scientists

There will be a strong cooperation with Prof. Dr. Helmut Herwartz from the Chair of Econometrics at the University of Kiel. Prof. Dr. Herwartz is an experienced econometrician with expertise on subjects such as forecasting with multivariate volatility models and their applications to Value-at-Risk. Prof. Dr. Herwartz has closely worked with Leonardo Morales-Arias in previous econometric projects. Another colleague with overlapping interests is Prof. Dr. Roman Liesenfeld (Chair of Statistics) who is a well-known expert in simulation-based estimation and stochastic volatility models. The researchers within this project will also work closely with Junior Prof. Dr. Simone Alfarano whose current research focuses on applications of statistical physics to economics and finance and the econometrics of high frequency data.

5.3 Collaboration with Scientists at other Universities

A close collaboration will be established with the group of Prof. Joachim Peinke (Chair of Hydrodynamics, University of Oldenburg). Prof. Peinke was one of the authors of the pioneering study on multi-scaling of financial returns that appeared in *Nature* (Ghasghaie et al.,1996). His group has recently developed an innovative non-parametric method for estimation of conditional probability densities at various scales from empirical data. In order to foster the communication and exchange between both teams, we plan to have annual meetings in Kiel and Oldenburg in the format of small workshops as well as one longer stay of the researchers-in-charge at Prof. Peinke's group. Part of the planned research will also be carried out in close cooperation with researchers abroad who have already been co-authors of previous publications on multifractal models: Prof. Tiziana di Matteo, Department of Applied Mathematics at Australian National University, Canberra and Ruipeng Liu, former Ph. D. student in Kiel and now Lecturer in Finance at Deakin University in Melbourne.

5.4 Research Capacities

The Chair of Monetary Economics and International Finance at the University of Kiel and the doctoral programme "Quantitative Economicsposses all the necessary resources to conduct the proposed research. With respect to data we have full access to *Datastream* which is a powerful compiler of financial time series. Moreover, we have a comprehensive CD-ROM collection of tic by tic data of the NYSE since December 1993 as well as similar data for the German stock market which can be used to compute realized volatility. We have access to GAUSS, MATLAB and EVIEWS licenses which are the most important softwares packages in applied research in finance and economics. There is direct access to the Central Library of Economics at the Kiel Institute for the World Economy which is one of the most complete libraries in economics in the world. The chair has also direct access to JSTOR and EBESCO for electronic journals.

6 Statement

A proposal for funding this project hasn't been submitted at any other office. If I should bring forward such a proposal, I will give notification to the *Deutsche Forschungsgemeinschaft* immediately.

7 Signature

(Prof. Dr. Thomas Lux)

8 List of Attachments

1. Curriculum vitae of Prof. Dr. Thomas Lux
2. Curriculum vitae of MSc Leonardo Morales-Arias
3. Curriculum vitae of MSc Payan Norouzzadeh

4. Publications on preliminary work for this project

9 Deutsche Version

Multifraktale Modelle von Finanzrenditen: Multivariate Erweiterungen, empirische Schätzung und Anwendung im Risikomanagment

Antrag auf Gewährung einer Sachbeihilfe eingereicht von:

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1.6. Zusammenfassung

Multifraktale Prozesse entstammen ursprünglich der stochastischen Physik und wurden dort zur Modellierung turbulenter Strömungen und verwandter Phänomene entwickelt. In jüngster Zeit fanden solche Modelle auch zunehmend in der empirischen Finanzmarktforschung Beachtung, da sie die wichtigsten Zeitreiheneigenschaften von Finanzrenditen abzubilden erlauben: Sowohl unterschiedliche Grade der zeitlichen Abhängigkeit verschiedener Schwankungsmaße wie auch die breiten Enden der Verteilung der Preisänderungen lassen sich mit multifraktalen Modellen aufgrund ihrer Konstruktionsprinzipien in robuster Weise generieren. Ein Nachteil des multifraktalen Apparats aus der Physik war jedoch die nicht-kausale Struktur der dort verwendeten Modelle, die auf kombinatorischen Zerlegungen für Wahrscheinlichkeitsmaße beruhten. Ein wichtiger Schritt zur Anwendung in der Finanzökonomik war daher die Entwicklung eines multifraktalen Zeitreihenmodells als Gegenstück zu den rein kombinatorischen Modellen der statistischen Physik. Obwohl die entsprechende Literatur sich noch in einem sehr frühen Stadium befindet, hat sie bereits eine ganze Reihe von Methoden der Inferenz für solche kausalen multifraktalen Modelle entwickelt und diese erfolgreich in empirischen Anwendungen (insbes. zur Vorhersage der Volatilität) getestet. Das vorliegende Projekt stellt sich die Aufgabe, die bisherigen Arbeiten zu multifraktalen Renditemodellen in verschiedener Hinsicht zu verallgemeinern und weiterzuentwickeln. Im Zentrum des Interesses wird dabei die Entwicklung multivariater Modelle stehen. Nach der Erprobung verschiedener Methoden der Parameterschätzung wird der Möglichkeit des praktischen Einsatzes multivariater Multifraktal-Modelle im Portfolio- und insbes. Risikomanagment im Vergleich zu etablierten Zeitreihenmodellen nachgegangen werden. Weitere Schwerpunkte werden die Modellierung der Innovationen in Relation zu der systematischen Komponente der Volatilitätsdynamik, die Anwendung auf Maße der sog. realisierten Volatilität und der Vergleich mit parameterfreien Multiskalierungsschätzern sein. Da multifraktales Verhalten viele Bereiche der Naturwissenschaft kennzeichnet, sollten unsere methodischen Fortschritte auch auf die ursprünglichen Anwendungsgebiete multifraktaler Modelle zurückwirken und etwa bei der Vorhersage von Niederschlagsmengen einsetzbar sein.

4 Benötigte Mittel

4.1. Personalkosten

Es wird die Bereitstellung einer 3/4-Stelle der Kategorie 13 TV-L für eine Dauer von drei Jahren beantragt. Eine 3/4-Stelle erscheint notwendig, um angesichts des hohen Gehaltsniveaus in nicht-akademischen Berufen für geeignet qualifizierte junge Wissenschaftler hinreichend attraktive Arbeitsbedingungen bieten zu können. Für das geplante Projekt stehen derzeit zwei hervorragende Kandidaten zur Verfügung: Während der ersten Stufe des Projekts soll die beantragte Stelle für etwa 1 Jahr mit Herrn Leonardo Morales-Arias (MSc in Economics der University of Cape Town) besetzt werden. Herr Morales-Arias ist derzeit fortgeschrittener Doktorand im Internationalen Doktorandenprogramm „Quantitative Economics" an der CAU. Er ist hervorragend ausgewiesen auf dem Gebiet der empirischen Finanzmarktforschung und besitzt insbesondere Erfahrung mit der Arbeit mit der ganzen Bandbreite traditioneller (uni- und multivariater) Volatilitätsmodelle. In der zweiten Phase wird die Verantwortung für die Projektarbeit auf Herrn Payam Norouzzadeh (MSc in Physics, Teheran University of Science and Technology) übergehen. Herr Norouzzadeh hat sich bereits in verschiedenen Veröffentlichungen mit der Anwendung des multifraktalen Apparates der stochastischen Physik auf Finanzmarktdaten beschäftigt und gleichzeitig für einige Jahre nach seinem Studium in der Finanzpraxis gearbeitet. Er hat zum Wintersemester 2007/08 ein Promotionsstudium im Programm „Quantitative Economics" aufgenommen und wird bis zu seiner Übernahme der Projektstelle das Kursprogramm des Doktorandenstudiums abgeschlossen haben. Aufgrund der Notwendigkeit, umfangreiche Routinearbeiten zur Vorbereitung der empirischen Anwendung unserer Modelle durchzuführen, wird zusätzlich die Finanzierung einer Stelle einer studentischen Hilfskraft beantragt. Die an der Universität üblichen Verträge für studentische Hilfskräfte sehen eine Arbeitsleistung von 30 Stunden /Monat vor. Die Beschäftigung von Hilfskräften dient auch zu deren Hinführung zu empirischer Arbeit mit Hochfrequenzdaten des Finanzsektors und wird in Zusammenhang mit Seminar- oder Diplomarbeiten organisiert werden.

4.2. Reisen

EUR 3000,- / Jahr. Der vorgesehene Betrag soll zum größten Teil für die Präsentation der wissenschaftlichen Ergebnisse des Projekts bei einschlägigen internationalen Tagungen verwendet werden. Dies wären insbes. die Jahrestagung der Society for Computational Economics, die jährliche Tagung “Forecasting of Financial Markets” sowie speziellere Konferenzen und Workshops im Bereich der Finanzökonomik und der „Ökonophysik“. Ein Teil der Reisemittel soll für ein jährliches Treffen mit der Gruppe von Prof. Peinke in Oldenburg zum Austausch über den Fortgang der jeweiligen Arbeiten verwandt werden, wobei die hierfür zu veranschlagenden Kosten wegen der räumlichen Nähe relativ gering sind (ca. EUR 500,- / Jahr)

4.3 Publikationskosten

EUR 750,- / Jahr

4.4. Sonstige Kosten

EUR 1500,- / Jahr. Dies beinhaltet einerseits einen Teil der zusätzlich anfallenden Verbrauchskosten (z.B. Druckermaterial), zum anderen notwendige Software, die zur Durchführung des Projekts zusätzlich anzuschaffen ist. Hierfür soll eine weitere Lizenz der Software GAUSS (inkl. verschiedener Module) beschafft werden. Die Kosten würden etwa hälftig aus dem laufenden Budget des Lehrstuhls und aus den Projektmitteln getragen.

6. Erklärung

Ein Antrag auf Finanzierung dieses Vorhabens wurde bei keiner anderen Stelle eingereicht. Wenn ich einen solchen Antrag stelle, werde ich die Deutsche Forschungsgemeinschaft unverzüglich benachrichtigen.

7. Unterschrift

8. Verzeichnis der Anlagen

1. Lebenslauf von Prof. Dr. Thomas Lux
2. Lebenslauf von MSc Leonardo Morales-Arias
3. Lebenslauf von MSc Payan Norouzzadeh
4. Veröffentlichungen zu Vorarbeiten für das Projekt

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