What Output-Capital Ratio to Adopt
For Macroeconomic Modelling?

Reiner Franke∗
University of Kiel (GER)

First Draft, February 2016

Abstract

Regarding the output-capital ratio in heterodox macroeconomic simulation studies, a surprisingly wide range of numerical values can be found. The paper discusses quarterly US data that are publicly available where, in order to capture depreciation, the construction of the capital stock by the perpetual inventory method relies on detailed estimates of its lifetime. Subsequently the paper builds up a capital stock series by alternatively having recourse to the statistics about capital consumption and furthermore determining an initial level by an assumption about the long-term growth of capital. This procedure leads to somewhat different results. In addition, the rates of depreciation and profit are studied that are implied by the two approaches. The paper closes with two numerical proposals for the steady state values of these variables and the output-capital ratio that could be readily employed for macrodynamic modelling, and that are quite different from many of the aforementioned examples.

JEL classification: C 13, E 12, E 30.

Keywords: Quarterly capital stock data, perpetual inventory method, Harrod-neutral technical change, capital consumption.

1 Introduction

A key variable in macrodynamic models linking stocks and flows is the output-capital ratio. If the reduced-form versions of these models have more than two or three state variables and one wants to analyse their properties in depth, one has to

∗Email: franke@uni-bremen.de.
make use of numerical simulations. Accordingly, one has to have an idea of the order of magnitude of that ratio. In this respect, however, a remarkable heterogeneity prevails and a very wide range of values are assigned to it in the literature. As a consequence, it is rather difficult to make comparisons across different models in order to get a comprehensive understanding of their similar and dissimilar features. Given the contemporary craft and skill in model building, this is an aspect where the profession still seems to be somewhat retarded.

It is in fact surprising that these studies hardly ever make reference to empirical evidence on output-capital ratios. Sure enough, the critical point is the data on a stock of fixed capital. The present contribution therefore begins by considering a series for the US economy that is not only conveniently available at a quarterly frequency but that is also well suited to the firm sector, i.e. non-financial corporate business, with which the great majority of the heterodox macroeconomic models are concerned. The series is obtained from the standard approach of the perpetual inventory method, which rests on three kinds of input: a level of the capital stock in a base period, data on gross fixed investment, and data or assumptions on depreciation.

After describing the construction of the capital stock series just mentioned, we will direct attention to another type of data that may proxy depreciation. In addition, we put forward a concept of how to determine the initial capital stock in an endogenous way. Its merit is that here only flow data are needed if one accepts the assumption of Harrod-neutral technical change (or some generalization of it) over a longer period of time. We will indeed be able to give some evidence on that. The capital stock series built up in this way will then deviate from the original one to a certain degree, and of course so will the corresponding output-capital ratios.

We can thus come up with two proposals of a numerical output-capital ratio that is ready for use, either as a typical value or as an entire series from 1960 on. For an informed assessment of these ratios, we will also have clarified the foundations from which they have been derived.

A first implication of the output-capital ratio is its impact on the level of the firms’ rate of profit, which plays an important role in many heterodox models. Since apart from capital depreciation the profit rate is given by the product of the profit share and the output-capital ratio, great differences in the latter give rise to great differences in the profit rate. Perhaps depending on the complexity of the models, this issue may not always be completely innocent in the simulations. At the end of our study we will also point out that here the relative price of fixed capital versus total production should not be neglected.

The remainder of the paper is organized as follows. The next section recapitulates the properties of the quarterly data that are readily available to compute output-capital ratios. While the construction of this capital stock rests on estimates
of the lifetime of the single capital items by the BEA (The Bureau of Economic Analysis), Section 3 proposes the statistics of capital consumption as an alternative on which to found a capital stock series. Section 4 turns to the rates of depreciation and profit implied by the two approaches. As an upshot of the analysis, we give a short summary of the specific numerical figures that macroeconomic models may adopt for their parameters and steady state values. This also includes a suggestion for a typical amplitude of the output-capital ratio in a cyclical framework. Section 5 concludes. The data source and a link for downloading the time series underlying our computations are given in an appendix.

2 The Fair-Parke capital stock series

It was mentioned in the Introduction that a great variety of numerical output-capital ratios can be found in the modelling literature. Measuring output at an annual rate, Table 1 presents a collection of eighteen recent examples (the references are listed in the appendix). Remarkably, only two of them make reference to empirical data. Most of this literature introduces their ratios even without any comment to motivate their choice.\(^1\)

<table>
<thead>
<tr>
<th>0.130</th>
<th>0.175</th>
<th>0.188</th>
<th>0.250</th>
<th>0.300</th>
<th>0.320</th>
<th>0.333</th>
<th>0.364</th>
<th>0.389</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.400</td>
<td>0.420</td>
<td>0.667</td>
<td>0.700</td>
<td>0.765</td>
<td>0.770</td>
<td>0.795</td>
<td>1.000</td>
<td>1.500</td>
</tr>
</tbody>
</table>

Table 1: A collection of numerical output-capital ratios from the modelling literature.

If one does not want to go to great pains with empirical issues, one can make use of a database of quarterly macroeconomic time series for the US economy that is freely provided by Ray C. Fair and William R. Parke (see the appendix for further details). It compiles data from the national income and product accounts and the flow of funds accounts, and also contains own computations of the authors. In the following this data set may be referenced as FP. All of the variables we are interested in are for the firm sector, i.e. essentially non-financial corporate business. A time series of the output-capital ratio in this sector is readily obtained by dividing

\(^1\)A further example is the book on medium-scale models by Charpe et al. (2011), in which the authors employ four different values for their output-capital ratio in a steady state position, namely, 0.50, 0.40, 0.45, 1.00 (on pp. 48, 109, 296, 303, respectively).
the (annualized) FP raw data series of total production (Y) by FP’s construction of a capital stock (end-of-quarter) of nonresidential fixed investment (KFP), both in real terms. The outcome is the bold (blue) line in Figure 1.

Asking for an equilibrium value of the output-capital ratio that may serve as a basis for numerical simulations, it remains to the researcher to choose a particular date from which to take Y/KFP, or a period over which an average value is computed. In any case, it is obvious that the majority of the values listed in Table 1 do not fall into that range.

Figure 1: FP series of an output-capital ratio in the firm sector.

FP derive their capital stock series from the estimates provided by the BEA (The Bureau of Economic Analysis). This data cannot be directly utilized because it is annual and concerned with total nonresidential investment in the economy, which besides the firm sector also covers the household, financial and government sector. The adjustment procedure is described in Fair (2013, pp. 358f). Letting KT be this larger capital stock, DEP its rate of depreciation per quarter, and IT total gross investment per quarter, the procedure starts out from the updating equation of the perpetual inventory method that links these quantities at a quarterly frequency,

$$KT_t = (1 - DEP_t)KT_{t-1} + IT_t$$

(1)
(that is, the length of this period is one quarter). Data from the BEA are quarterly available for IT and at the end of each year for KT. On this basis, FP determine DEP residually from (1) under the assumption that the quarterly rates remain constant over a year. Thus, with respect to \( t \) being the first quarter of a year and \( KT_{t-1} \), \( KT_{t+3} \) the available BEA capital stocks, the value of \( DEP_{t+3} = DEP_{t+2} = DEP_{t+1} = DEP_t \) is computed such that it matches eq. (1) over the four quarters with these initial and terminal capital stocks.

In this way one also gets a quarterly interpolation of the BEA stocks. It remains to downscale them to the size of the firm sector, which we then designate KFP. To this end, FP adopt their results for \( DEP_t \), turn to nonresidential fixed investment \( IF_t \) in the firm sector, and reconsider (1) in the form

\[
KFP_t = (1 - DEP_t) KFP_{t-1} + IF_t \tag{2}
\]

This series is established once a value for KFP in a base quarter is chosen. For 1952:1, FP put it equal to a fraction 0.85831 of KT. This value is the average share of IF in total investment IT over the period from 1952:1 to 2015:1,

\[
KFP_t = \tilde{\phi} KT_t, \quad \tau = 1952:1 \text{ and } \tilde{\phi} = \text{average } \{IF_t/IT_t\}_{t=1952:1}^{2015:1} \tag{3}
\]

The ratio \( IF_t/IT_t \) is not constant over time but shows some trending behaviour. Certain intermediate fluctuations apart, it decreases from 89.8% in 1960:1 to 81.3% in 1989:3, increases to 84.5% in 1995:3, decreases to 79.5% in 2004:2, and increases again to 83.5% in 2015:1. As a supplement to (3), one may seek to take these changes in the structure of fixed investment into account. In order to see if this idea may lead to a sizeable effect on the output-capital ratio, the end-of-year values of the firms’ capital stock KFP may be obtained as time-varying proportions of KT, where it is most straightforward to employ the contemporary investment proportions. Accordingly, let us consider a modified capital stock series KFP_{mod} that uses the proportions \( \phi_t = IF_t/IT_t \) to determine the stocks in the firm sector from the interpolated total capital stocks KT as KFP_{mod, t} = \phi_t KT_t. Hence this series is related to KFP from (2) and (3) as

\[
KFP_{mod, t} = (\phi_t/\tilde{\phi}_t) KFP_t, \quad \phi_t = IF_t/IT_t \tag{4}
\]
The corresponding output-capital ratios are shown as the thin (red) line in Figure 1. It demonstrates that the differences from the original ratios are negligible until the beginning of the 1980s, but that afterwards $Y/KFP_{mod}$ tends to exhibit values that exceed $Y/KFP$ by a margin of roughly 0.10. Of course, it depends on the context in which such output-capital ratios are applied whether these deviations would be reckoned to be significant or only of secondary importance.

3 An alternative concept to build a capital stock series

In the macroeconomic framework of the flow of funds accounts, capital depreciation is reflected by the capital consumption statistics. Such a series for the firm sector is also available from the FP database and may be designated CCF (for real capital consumption in the firm sector, per quarter). In order to check the implications of the capital stock data $KFP$ and its depreciation rates $DEP$ from the previous section, it seems natural to compare the wear and tear predicted by this approach to the CCF data. Normalizing the level variables by the (quarterly) output in the firm sector, let us therefore consider the following two concepts:

\[
\begin{align*}
\text{DPY-FP}_t &= 100 \cdot DEP_t \cdot KFP_{t-1} / (Y_t/4) \\
\text{DPY-CC}_t &= 100 \cdot CCF_t / (Y_t/4)
\end{align*}
\]

While ideally the series should move fairly close to together, the upper panel of Figure 2 demonstrates systematic deviations from one another. Depreciation from $KFP$ is lower than inferred from CCF over the first three decades of the sample, which might indicate that the BEA data have somewhat underestimated the factor of economic obsolescence. The differences disappear over the next ten years and the relationship is reversed after the first few years of the new century, where the role that a little bit later the financial crisis possibly might have played is an open question. Generally, depreciation exhibits a clear upward trend until at least 2000.

The differences in the two series of eq. (5) have an effect on the evolution of net investment over time. Accordingly, the increases in the BEA capital stock will have been stronger over the first thirty years after 1960, and weaker over the last ten years, than those of a capital stock with capital consumption CCF. The relative size of these effects and the impression they have on the naked eye will depend on the level of gross investment $IF$, its possibly changing long-run behaviour, and its variations over the business cycle. The two net investment-to-output ratios resulting
Figure 2: The ratios of depreciation to output from eq. (5), and of net investment to output from eq. (6).

from (5) are presented in the bottom panel of Figure 2, where the two series are given by the following data:

\[
\begin{align*}
\text{NIY-FP}_t &= 100 \cdot \left( KFP_t - KFP_{t-1} \right) / (Y_t/4) \\
\text{NIY-CC}_t &= 100 \cdot \left( IF_{t-1} - CCF_t \right) / (Y_t/4)
\end{align*}
\]  

(6)

Normalized by output, net investment in the KFP capital stock shows a declining tendency. This is evidenced by the downward-sloping dotted (blue) line, which is obtained from a simple linear regression of NIY-FP on time. To this end the sample was terminated in 2007:2 to rule out the severe effects originating with the financial crisis. By contrast, when considering the period from 1960:1 to 2007:2 in its entirety, the net investment ratio NIY-CC generated by the capital consumption statistic shows no systematic trend; see the thin solid (red) regression line, the slope of which is practically zero. The average ratio NIY-CC is thus equal to 3.106.
The latter observation is a welcome feature from a theoretical point of view. In practically all heterodox macrodynamic models the output-capital ratio is a constant magnitude in a steady state position, that is, the assumption of Harrod-neutral technical change prevails. Consequently, also net investment and output grow in step; or if such a model exhibits cyclical behaviour around a long-run equilibrium, the net investment-to-output ratio will fluctuate around a constant value. Hence, if we abstract from the possible weak upward trend in the first twenty years of the sample and a weak downward trend in the subsequent twenty years, the empirical series NIY-CC\textsubscript{t} in Figure 2 is compatible with Harrod neutrality.

A constant output-capital ratio in a steady state is tantamount to equal growth rates of output and capital, while in a cyclical framework the statement applies to the long-run time averages. This idea can be utilized to build a capital stock series with again the aid of the perpetual inventory method, which, however, now employs the capital consumption data to capture depreciation. The point is that the growth rates of such a series will depend on the level of the capital stock in a base period. Therefore, if the series is supposed to exhibit Harrod neutrality, we can determine the initial level endogenously, so to speak, such that the resulting average growth rate of capital will be equal to the average growth rate of output (which is a given magnitude).

Denote these alternative capital stocks by KCC\textsubscript{t} (the suffix ‘CC’ may be indicative of constructions where empirical depreciation is represented by capital consumption). Formally, over the period from \( t = 1 = 1960:1 \) to \( t = T = 2007:2 \) the rule just described reads,

\[
\begin{align*}
  KCC\textsubscript{t} & = KCC\textsubscript{t-1} - CCF\textsubscript{t} + IF\textsubscript{t}, \quad t = 1, \ldots, T \\
  KCC\textsubscript{0} & \quad \text{such that } \text{av}(gKCC) = \text{av}(gY)
\end{align*}
\]

(7)

where \( gX\textsubscript{t} \) designates the (annualized) growth rates of a variable \( X\textsubscript{t} \) between quarter \( t-1 \) and \( t \), and \( \text{av}(gX) \) is their average over the sample \([1, T]\).\(^5\) It is obvious that the capital growth rates are a decreasing function of the initial level \( KCC\textsubscript{0} \), such that the average growth rate of KCC can be made arbitrarily small by choosing \( KCC\textsubscript{0} \) sufficiently low and vice versa. Hence there will a unique solution of \( KCC\textsubscript{0} \) that brings about the desired equality in (7). As an advantage of this construction device it may also be noted that it only requires very elementary macroeconomic flow data, provided one is willing to accept the hypothesis of on average Harrod-neutral technical change.\(^6\)

\(^5\)The average growth rate \( g \) of a variable \( X \) over a period of \( T \) quarters is determined as the solution to the equation \( X\textsubscript{T} = (1 + g)^T X\textsubscript{1} \).

\(^6\)In principle, the approach of eq. (7) could also be applied if there is evidence of another type of technical change from which one can derive growth rate differentials between capital and output over a certain span of time.
Before turning to the computation of (7), we can do a little back-of-the-envelope calculation to get a feeling for the order of magnitude of the thus resulting output-capital ratio. To this end let NI-CC, stand for the level of quarterly net investment in (6) and consider the identity

\[
\text{NIY-CC}_t = \frac{\text{NI-CC}_t}{Y_t/4} = 4 \cdot \frac{\text{NI-CC}_t}{\text{KCC}_t} \cdot \frac{\text{KCC}_t}{Y_t} = g_{\text{KCC}} \cdot \frac{\text{KCC}_t}{Y_t} \quad (8)
\]

As noted above on Figure 2, the average net investment-to-output ratio between 1960:1 and 2007:2 on the left-hand side of (8) is equal to 3.106. Regarding the last term, roughly the same should hold for the division of the average capital growth rate by the mean value of the output-capital ratio. By hypothesis, the former is equal to the average output growth rate, for which we obtain 3.464%. In sum, by “solving” (8) for Y/KCC the average output-capital ratio is approximately determined as

\[
\text{av} \left[ \frac{Y}{\text{KCC}} \right] \approx \frac{\text{av}(g_{\text{KCC}})}{\text{av}[\text{NIY-CC}_t]} = \frac{\text{av}(g_Y)}{3.106} = \frac{3.464}{3.106} = 1.115 \quad (9)
\]

As the output-capital ratios Y/KFP in Figure 1 fluctuate considerably below this level, eq. (9) tells us that at least most of the time the alternative capital stocks KCC, will be lower than the ones from FP. In fact, the initial level of KCC in (7)

\[
\text{KCC}_0 = 0.749645 \cdot \text{KFP}_0
\]

Between 1960 and 1973 the difference between KCC and KFP increases from 25% to 31%, which is explained by the fact that net investment NIY-CC falls short of NIY-FP, during that period. Over the next thirty years the investment differences are so small that the lower levels of KCC let these capital stocks increase at higher rates than KFP. This steadily reduces the gap between KCC and KFP from 31% to 21% in 2003. As seen in Figure 2, in the last ten years NIY-CC exceeds NIY-FP substantially. As a consequence, the gap narrows more rapidly. Finally in 2015, KCC turns out to be practically equal to KFP.

The corresponding output-capital ratios are shown in Figure 3; the bold (blue) line depicts Y/KCC, while the lower thin (blue) line reproduces the series Y/KFP from Figure 1. What perhaps first leaps to the eye is the dramatic fall of Y/KCC caused by the Great Recession, whereas the change in Y/KFP looks relatively moderate. The dissimilarity is, of course, due to the weak growth in KFP, evidenced in the bottom panel of Figure 2 as opposed to the still notable growth of KCC, which outdistances output growth over the last ten years by far.

Over the main sample period until 2007:2, however, the ratios Y_t/KCC_t exhibit no comparable upward or downward trend. By construction, the initial and
terminal ratios coincide, at a level of $Y_1/KCC_1 = Y_T/KCC_T = 1.061$. On average, the output-capital ratio is a bit higher, $\text{av}[Y/KCC] = 1.104$, which is fairly close to the rough estimate from eq. (9).

At the end of this section, the contribution of the different depreciation concepts in the perpetual inventory method may be worked out more clearly. We may thus start the mechanism for the capital stocks in the first line of (7) at the same level as FP. That is, distinguishing these capital stocks from KCC by a superscript ‘(1.00)’, we deviate from (10) and set $KCC_{(1.00)}^0 = 1.00 \cdot KFP_0$, so that the series $KCC_{(1.00)}^t$ differs from $KFP_t$ only by the depreciation term in the updating procedure: $CCF_t$ in (7) versus $\text{DEP}_t \cdot KFP_{t-1}$ in (1). Because, as seen in the top panel of Figure 2, the depreciation-to-output ratios $DPY-CC_t$ exceed $DPY-FP_t$ over the first half of the sample, the capital stocks $KCC_{(1.00)}^t$ are here increasingly lower than $KFP_t$. Correspondingly, the output-capital ratios $Y_t/KCC_{(1.00)}^t$ are increasingly higher than the ratios $Y_t/KFP_t$; see the thin (red) line in the middle of Figure 3. Our alternative proposal of an output-capital ratio $Y_t/KCC_t$ is certainly still higher since the stocks $KCC_t$ start out from a lower initial level.

Figure 3: Output-capital ratios from (7) and (10).
4  Implied depreciation and profit rates

In many macroeconomic growth cycle models the output-capital ratio is a central dynamic variable in (at least) two respects. It measures the fluctuations of economic activity around a long-run equilibrium growth path, and it co-determines the level and the fluctuations of the firms’ rate of profit (which in turn has an effect on aggregate demand and features in the financing of investment). In a one-good world, the latter is commonly specified as

\[
\tilde{r} = (1 - \tau)(1 - \omega)y - \delta
\]  

(11)

Here \(\tilde{r}\) denotes the profit rate (a tilde is added above the usual symbol \(r\) because in a moment a little modification of (11) will be introduced), \(\tau\) is a tax rate on production (mostly zero for simplicity), \(\omega\) the share of wages in gross value added after these taxes, \(y\) the output-capital ratio, and \(\delta\) the rate of capital depreciation. The determination of taxes and income distribution is beyond the scope of the present contribution. On the basis of the discussion of the output-capital ratio in the previous section, however, we want to get a feeling for the order of magnitude of the rate of profit, which also requires knowledge of the depreciation rate entering it.

Let us therefore begin with considering the values for \(\delta\). The upper panel in Figure 2 has already shown the differences in the two depreciation concepts that we considered, While there output served as a normalization factor, we can now divide the depreciation variables by the corresponding capital stocks and thus obtain their rates of depreciation. Annualized and expressed in per cent, these are the two series in the top panel of Figure 4. The bold (blue) line series designated DEP-CC depicts the depreciation rates derived from the capital consumption statistics and the capital stock \(KCC\), i.e. \(DEP-CC_t = \frac{CCF_t}{KCC_t} - 1\). The thin (red) line represents the depreciation rates that FP adopted from the BEA, i.e. \(DEP-FP_t = DEP_t\) in the notation of Section 2.

The overall pattern of the two depreciation rates is similar to Figure 2, but now we also have concrete numerical magnitudes that can be used by models to calibrate their \(\delta\). Both rates are increasing until at least the turn of the century, where they still remain below the level of a common benchmark like ten per cent per year. The ups and downs of \(DEP-CC_t\) after 2000 are probably not only due to technological factors or economic obsolescence but also to special (temporary) legislative acts (such as the per cent of bonus depreciations that businesses are allowed to claim, for example). We need not bother about these details since our main interest attaches to the general level of a depreciation rate, which in a modelling context will be constant anyway.

Turning to the rate of profit, eq. (11) deserves a second look. Decomposing aggregate output in a closed economy into gross investment on the one hand and...
Figure 4: Empirical proxies for the depreciation rates $\delta$, relative prices $p_{yk}$, and hypothetical rates of profit $r$.

Note: See text for the construction of the series.

private and public consumption on the other hand, there are in general three price levels to consider: the price $p_y$ of total output $Y$, the price $p_k$ of the capital goods $K$, and the price of the consumption goods. With this differentiation the aggregate profit rate $r$ reads,

$$ r = \frac{(1 - \tau)(1 - \omega) p_y Y - \delta p_k K}{p_k K} = \frac{(1 - \tau)(1 - \omega) p_{yk} y - \delta}{p_{yk}} := \frac{p_y}{p_k} $$

(12)

Definition (11) takes the assumption of a one-good economy literally in that it postulates identical prices for all goods, $p_{yk} \equiv 1$. Empirically, however, there is neither a tendency for similar levels of the corresponding price deflators nor that $p_y$ and $p_k$ move in step. The middle panel of Figure 4 with the time series $p_y/p_k$ shows
that around 1960 one unit of capital could almost buy two units of total output, and that over time capital was in this respect steadily cheapening. Nevertheless, even in 2015 its price level was still higher than \( p_y \).

This observation prompts us to abandon the universal modelling hypothesis \( p_{yk} \equiv 1 \). In fact, eq. (12) makes us aware that the aggregate profit rate does not only vary with depreciation, the wage share and capital utilization \( Y/K \), but also with relative prices. More specifically, it will rise if the investment goods tend to become relatively cheaper over time.

To get an impression of the latter effect and, of course, the numerical level of the profit rate \( r \) in general, we fix taxes (\( \tau \)) and income distribution (\( \omega \)) and apply our empirical and time-varying counterparts of \( y \) and \( \delta \) to (12). Regarding the former, let us borrow from Franke (2015) the (rounded) empirical time averages of \( \tau \) and \( \omega \) over the years 1983:1 – 2007:2. This gives us \( \tau = 8.50\% \) and \( \omega = 79\% \), so that we may work with \((1 - \tau)(1 - \omega) = 0.28\). Using the two depreciation and capital stock concepts from above and denoting by PYK the empirical price ratio \( p_y/p_k \), we thus take into account the series of the two hypothetical profit rates,

\[
\begin{align*}
\text{RFP}_t &= 0.28 \cdot \text{PYK}_t \cdot \frac{Y_t}{KFP_t} - \text{DEP-FP}_t \\
\text{RCC}_t &= 0.28 \cdot \text{PYK}_t \cdot \frac{Y_t}{KCC_t} - \text{DEP-CC}_t
\end{align*}
\]

The bottom panel of Figure 4 shows the outcome. Clearly, \( \text{RCC}_t \) must be persistently above \( \text{RFP}_t \) because we know from Figure 3 that the same holds for the corresponding output-capital ratios, while the mostly positive differences between the depreciation rates \( \text{DEP-CC}_t \) and \( \text{DEP-FP}_t \) are not large enough to offset this effect.

Until the beginning of the 1980s the evolution of the profit rates is primarily determined by the associated output-capital ratios. Correspondingly, if anything, there is a weak downward trend in the profit rates over this period. Over the next twenty years until 2000 both output-capital ratios are mainly rising. This effect on the profit rates is reinforced by the relative prices \( \text{PYK}_t \), the increase of which is accelerating then. The possibly counteracting effects from the rising depreciation rates are again too weak to make themselves felt. As a result, the profit rate \( \text{RFP}_t \) increases from a trough value of 8.1% in 1982:3 to to a peak of 14.3% in 2000:1, while \( \text{RCC}_t \) increases from 10.6% in 1982:4 to 18.3% in 2000:3. Note that these rates would be significantly higher if, with the definition of (11), the relative prices were ignored.

When our discussion of the data is considered sufficiently serious and it comes to a numerical calibration of a model on this basis, it will depend on the researcher what historical episodes he or she chooses to focus on. To come up with a succinct summary, we may propose to refer to the period of the Great Moderation
(GM) from, say, 1980:1 until 2007:2 and adopt the time averages of the variables as constant parameters or steady state values, respectively.

In particular, in order to avoid the complications arising from a full-fledged two-good economy, the price ratio \( p_{yk} \) may be treated as an exogenous constant, which amounts to postulating identical inflation rates for \( p_y \) and \( p_k \). If one attaches importance to equal profit rates in the capital and consumption good sectors, this feature could be achieved by a suitable setting of the corresponding output-capital ratios, of which \( y \) would be a weighted average. If in a cyclical setting the composition of total output varies, \( y \) would undergo certain variations, too, even if the sectoral ratios did not. This effect is absent in (12), that is, strictly speaking the equation is an approximation. However, such an aggregation problem applies to all macroeconomic modelling that would not take the one-good story literal.

\[
(1 - \tau)(1 - \omega) \quad p_y/p_k \quad \delta \quad y^* \quad r^* \quad y = y^* \pm \quad r = r^* \pm
\]

<table>
<thead>
<tr>
<th></th>
<th>Steady State Values</th>
<th>Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP:</td>
<td>( 0.28 )</td>
<td>( 0.747 )</td>
</tr>
<tr>
<td></td>
<td>( \pm 0.016 )</td>
<td>( \pm 0.029 )</td>
</tr>
<tr>
<td>CC:</td>
<td>( 0.28 )</td>
<td>( 0.747 )</td>
</tr>
<tr>
<td></td>
<td>( \pm 0.038 )</td>
<td>( \pm 0.79% )</td>
</tr>
</tbody>
</table>

Table 4: Example of a numerical calibration.

The first three columns of Table 4 report the time averages of \( (1 - \tau)(1 - \omega) \), which have already been mentioned, the price ratios \( p_{yk} = p_y/p_k \), as well as of the depreciation rates \( \delta \) and the output-capital ratios \( y^* \) that we obtain from FP and CC, respectively; the star symbol may indicate that regarding modelling these values are viewed as prevailing in a steady state position.\(^7\) The equilibrium rates of profit \( r^* \) in the fourth column to which they give rise can then be computed from eq. (12).

One might wonder that the profit rate based on FP is perhaps too low, because after paying corporate taxes at a rate that averages around 28\%, it is reduced to

\[^7\text{Against the background of Table 4 and the literature overview in Table 1 above, it is interesting to note that for the estimation of their Kaleckian saving and investment functions Hein and Schoder (2011, p. 702) report an average output-capital ratio of 0.8028 over 1983 – 2007 (a value that in a later paper Schoder (2014) himself takes no account of). Here capital is the real net capital stock of the business sector, output is real net domestic product, and the authors’ data source is the OECD Economic Outlook and the European Commission’s Directorate General for Economic and Financial Affairs (cf. pp. 722f).}\]
8.43%. Modelling details apart, this figure is a first indicator of the returns to the shareholders, which thus (before present-day monetary policy) seem rather close to interest rates incurring a lower risk than the long-term investment in real capital. This observation may be a reason to have a preference for the CC approach and its implications.

With a view to models studying cyclical dynamics, some information can be added about the order of magnitude of typical fluctuations of output and profits. Thinking of deterministic models generating quite regular oscillations of $y$ and $r$, we want to know how much their peak and trough values should deviate from the equilibrium levels $y^*$ and $r^*$. To this end, we first consider the time series of the output gap over GM, i.e. the percentage deviations of $Y$ from a trend line $Y^*$. It is convenient to employ the Hodrick-Prescott filter for that purpose. Using the standard smoothing parameter $\lambda = 1600$ for quarterly data, we compute 1.39% for the standard deviation of the output gap. As it may be argued that this $\lambda$ does not yet sufficiently remove the business cycle from the trend, we alternatively propose to adopt a higher $\lambda = 80000 = 50 \cdot 1600$. This device gives us higher/deeper peaks and troughs. Numerically, the standard deviation of the gap increases to 2.46%.

In a deterministic setting where for a rough-and-ready evaluation of the fluctuations we may refer to a sine wave motion with a period of 8.50 years, these standard deviations are produced by an amplitude of $\pm 1.96\%$ and $\pm 3.47\%$ around the zero level, respectively. Let us round them off at $\pm 2.00\%$ and $\pm 3.50\%$. In a model with an equilibrium output-capital ratio $y^*$, the output gap is equal to $(Y - Y^*)/Y^* = (y - y^*)/y^*$. Hence the amplitude of the oscillations of $y$ is given by $y = y^* \pm y^* \cdot \text{maximal gap}$. The second-to-last column of Table 4 reports what this means for our values of $y^*$, while the last column computes the resulting amplitudes of the profit rates.

The amplitudes in Table 4 may appear somewhat low; lower at least than many numerical simulations in the literature suggest (when their output-capital ratios are rescaled to the present levels). The amplitudes will widen if also variations of the wage share over the cycle are taken into account, but this effect will presumably be fairly limited. If the results collected in Table 4 are basically accepted, we can point out an immediate consequence for the calibration of models that emphasize the role of profitability: the components of aggregate demand influenced by profit income will have to exhibit a relatively high sensitivity to that variable.

---

8With respect to the corporate tax rate we may refer to the computation in Franke (2015, Table 2 and the Appendix). The prime rate averages close to 8% over GM, and the AAA (BAA) bond rates are slightly lower (somewhat higher).

9There are other effects on the output gap around the zero line. They are, however, more ambiguous, so that we have no clear pros and cons for either of the two options.
5 Conclusion

Given that the output-capital ratio is a central magnitude in numerical studies of models linking stocks and flows, the paper started out from the observation that an extremely wide range of such values can be found in the heterodox literature, and that generally not much effort is expended to motivate them. We drew attention to a macroeconomic data source provided by Fair and Parke (FP) that could be quite conveniently consulted for that purpose. The key variable is, of course, the capital stock in the firm sector. We recapitulated how FP constructed a quarterly series from BEA data by invoking the perpetual inventory method. An essential point is here the depreciation data, which the BEA derived from its estimates of the lifetime of a large number of different capital items.

As an alternative, we considered the statistics about capital consumption to proxy depreciation. The capital stock in a base period for the perpetual inventory method was furthermore endogenously determined such that the average of the thus implied capital growth rates equals that of output growth, a feature for which some evidence was obtained before. The output-capital ratios resulting from this approach are systematically higher than those from FP. Nevertheless, it should be mentioned that the majority of the references from the literature do not even come close to either of these figures.

The paper concluded with a summary of two sets of a numerical output-capital ratio as well as depreciation and profit rates that are ready for use in macroeconomic simulations. We do not claim that one is superior to the other. It was just indicated that the one yielding the higher rate of profit may be preferred because its greater difference from the empirical level of interest rates appears more reasonable. In any case, readers and researchers might become more sensitive to numerical issues around the output-capital ratio and their justification, which after all should be no big problem.

Appendix

Examples of output-capital ratios from the literature

Regarding the examples of the numerical output-capital ratios listed in Table 1, the following Table A1 gives the precise references. The only examples we are aware of that have an explicit recourse to empirical data are Flaschel et al. (1997, pp. 317, 420) and Chiarella et al. (2005, p. 85). Some of the references in Table A1 do not make the underlying time unit or the length of their adjustment period in discrete time explicit. In these cases we checked as far as this was possible that the
interpretation of an annual rate is compatible with reasonable values for some other variable(s) in the model, such as an inflation or interest rate, in particular.

### The data source

Out data source is the database `fmdata.dat` in the zip file `fmfp.zip` that is provided by Ray Fair on his homepage for working with his macroeconometric model, on [http://fairmodel.econ.yale.edu/fp/fp.htm](http://fairmodel.econ.yale.edu/fp/fp.htm). This is a huge plain text file from which the single time series have to be extracted for further use. Each of them is identified by an acronym. They are explained in Appendix A.4, Table A.2. on pp. 190ff, of the book *Estimating How The Macroeconomy Works* by R.C. Fair, January 2004, which can be downloaded from [http://fairmodel.econ.yale.edu/rayfair/pdf/2003APUB.pdf](http://fairmodel.econ.yale.edu/rayfair/pdf/2003APUB.pdf) (last accessed December 2015).

The following table lists the FP time series used in this paper. The flows are per quarter if not indicated otherwise. Investment is gross investment, ‘f’ refers to

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.130</td>
<td>Le Heron and Mouakil (2008)</td>
</tr>
<tr>
<td>0.175</td>
<td>Nikolaidi (2014)</td>
</tr>
<tr>
<td>0.188</td>
<td>Skott and Ryoo (2008)</td>
</tr>
<tr>
<td>0.250</td>
<td>Asada (2012)</td>
</tr>
<tr>
<td>0.300</td>
<td>Chiarella and Di Guilmi (2014)</td>
</tr>
<tr>
<td>0.320</td>
<td>Ryoo (2015)</td>
</tr>
<tr>
<td>0.333</td>
<td>Grasselli and Costa Lima (2012)</td>
</tr>
<tr>
<td>0.364</td>
<td>Charpe et al. (2014)</td>
</tr>
<tr>
<td>0.389</td>
<td>Sordi and Vercelli (2014)</td>
</tr>
<tr>
<td>0.400</td>
<td>Ryoo (2010)</td>
</tr>
<tr>
<td>0.420</td>
<td>Schoder (2014)</td>
</tr>
<tr>
<td>0.667</td>
<td>Chatelain (2010)</td>
</tr>
<tr>
<td>0.700</td>
<td>Chiarella et al. (2005)</td>
</tr>
<tr>
<td>0.765</td>
<td>Charpe et al. (2012)</td>
</tr>
<tr>
<td>0.770</td>
<td>Pedrosa and Macedo e Silva (2014)</td>
</tr>
<tr>
<td>0.795</td>
<td>Dos Santos and Zezza (2008)</td>
</tr>
<tr>
<td>1.000</td>
<td>Charpe et al. (2015)</td>
</tr>
<tr>
<td>1.500</td>
<td>Caiani et al. (2014)</td>
</tr>
<tr>
<td>1.500</td>
<td>Flaschel et al. (1997, Ch. 11)</td>
</tr>
</tbody>
</table>

**Table A1**: A collection of numerical output-capital ratios from the modelling literature.
the firm sector (non-financial corporate business). This data can be directly downloaded in a text file from http://www.gwif.vwl.uni-kiel.de/de/working-papers-1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>FP acronym or construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Production in the firm sector per annum</td>
<td>4 · Y</td>
</tr>
<tr>
<td>KFP</td>
<td>Fixed capital stock in f</td>
<td>KK</td>
</tr>
<tr>
<td>DEP-FP</td>
<td>Depreciation rate of KK</td>
<td>DEL</td>
</tr>
<tr>
<td>IF</td>
<td>Real fixed nonresidential investment in f</td>
<td>IKF</td>
</tr>
<tr>
<td>IT</td>
<td>Total fixed nonresidential investment</td>
<td>IKF+IKH +IKB+IKG</td>
</tr>
<tr>
<td>PF</td>
<td>Price deflator for non-farm sales</td>
<td>PF</td>
</tr>
<tr>
<td>PK</td>
<td>Price deflator for capital goods</td>
<td>PIK</td>
</tr>
<tr>
<td>—</td>
<td>Nominal capital consumption in f</td>
<td>CCF1</td>
</tr>
<tr>
<td>CCF</td>
<td>Real capital consumption</td>
<td>CCF1/PIK</td>
</tr>
</tbody>
</table>

Table A2: Time series from the FP database.

References


