Microfounded Animal Spirits
and Goodwinian Income Distribution Dynamics

Reiner Franke*
Department of Economics
University of Kiel
Kiel, Germany
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Abstract
The paper considers the formation of an average opinion index in a microfounded
framework where the agents switch between two kinds of sentiments with certain
transition probabilities. The index can thus represent a general business climate,
or the famous animal spirits. Circumventing the elaborated tools of statistical
mechanics that are usually here applied, a more elementary argument is put
forward that allows one to derive a macroeconomic adjustment equation for
the climate variable, which is also shown to contain a global self-stabilization
mechanism. Combining this building block with a simple multiplier and a real
wage Phillips curve, a structurally stable model of Goodwinian growth cycles
with a herding component is obtained.

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*Correspondence: franke@iksf.uni-bremen.de. Financial support from EU STREP ComplexMarkets,
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1 Introduction

While rational expectations are the ruling paradigm in macroeconomic theory, expectations in the real world are far more diffuse and do not only concern just one variable such as typically the rate of inflation. Apart from that, expectations are not homogeneous as they are in these models where everything boils down to the representative agent. Insofar as expectations enter the agents’ economic decisions, a more appropriate concept appears to be that of an aggregate climate index, which still allows the individual agents to have different attitudes and make different decisions. If here the microeconomic basis is made explicit and a procedure is given of how to derive a macroeconomic adjustment equation for the climate variable, then from the viewpoint of heterodox theory this would be a more satisfactory approach to the microfoundations of macroeconomics and the treatment of expectations than the optimizing framework where all agents are blessed with the same rational expectations.

A route in this direction has been contrived more than twenty years ago in a stimulating book in the social sciences by Weidlich and Haag (1983). Unfortunately their approach has not found its way into contemporary macroeconomic theory. As we see it, two reasons are responsible for this neglect. First, the formulation of the model did not only refer to a probabilistic framework, its analysis also utilized concepts from the theory of statistical mechanics like the Master equation and the Fokker-Planck equation, which are largely unknown to many economists. In their work they usually study trajectories of definite aggregate variables, whereas these equations are concerned with the evolution of an entire probability distribution or at least, in the mean field approximations, with the time path of expected values. Even the latter concept, however, can be hard to assess, namely, if the stochastic equilibrium of the system is—most interestingly—characterized by a bimodal (or multi-modal) probability density function, in which case expected values would become meaningless in predicting the likely value of a variable.

A second aspect is insufficient marketing. Although the approach was employed in a number of macroeconomic papers, the topics they dealt with were somewhat detached or “exotic” (Kraft et al., 1986; Haag et al., 1987; Weise and Kraft, 1988), or the ordinary reader probably soon drowned in a sea of specification details so that he or she could no longer appreciate the essence of the approach (Weidlich and Braun, 1992; despite the promising
title of this paper). Nevertheless, macroeconomists with a wider area of interest could have also learned from several related articles in highly reputable journals, in particular by Kirman (1993), Orléan (1995), or Lux (1995, 1997, 1998). In sum, the approach by Weidlich and Haag (1983) or similar formulations in the 1990ies offered macroeconomists a good chance to introduce microfounded expectation formation into their models in a very convenient, even standardized way, but this chance was largely missed.1

Referring to the fields of macroeconomics, the present paper can be seen as providing a better marketing for the basic idea developed by Weidlich and Haag. The main innovation in this respect is the derivation of an ordinary adjustment equation for the realized values of an aggregate climate index, which moreover can be done without any recourse to the statistical mechanics apparatus.2 This macroeconomic equation is based on the concept of state-dependent transition probabilities that govern the switches of the individual agents between optimism and pessimism, for which we postulate an element of hetero-reference and self-reference. It may here be emphasized that the latter can also be described as giving rise to herding. The resulting design is in fact well compatible with Keynes’ discussion of “The state of long-term expectation” in Chapter 12 of the General Theory, so that the often quoted “animal spirits” and their endogenous evolution over time are given a more precise meaning.3

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1Taylor and O’Connell (1985), Franke and Asada (1994), and Flaschel et al. (1997, Chapter 12) are three of the few macrodynamic contributions whose central expectational variable is an economy-wide business climate, which is there called a state of confidence. The dynamic adjustments of the latter, however, were formalized in an ad-hoc manner. Without essentially affecting the final results, this part of the models could be easily, and conceptually more satisfactorily, reformulated along the lines propounded in the present paper. Hence, the implicit criticism of not having been sufficiently alert to a fruitful and innovative idea in the past also falls back on the author of this paper, especially since he knew of the articles by Weise and Kraft (1988) and Lux(1995) already quite early.

2However, the paper cannot claim the copyright on this procedure. The main arguments have been put forward in a recent article by Alfarano and Lux (2005) on an agent-based stochastic volatility model for financial markets, where the authors use them to discuss some simulation details (in Appendix A1 and A2).

3Regarding the expression of “animal spirits” in the macroeconomic literature a note of caution is in order. In the dynamic stochastic general equilibrium models of mainstream theory, animal spirits are synonymous to sunspot equilibria (which most of these macroeconomists view as a theoretical curiosity). This is a situation where equilibrium is indeterminate and endogenous persistent fluctuations are generated by ‘coordinated’ (and thus self-fulfilling) revisions of expectations (since, technically, agents can be conceived...
After delineating the way from the micro to the macro level, we present an elementary (and non-exotic) macroeconomic application. Interpreting the climate variable as a business climate determining fixed investment of firms and combining it with a simple multiplier theory and a wage Phillips curve, a two-dimensional differential equations system is obtained that can be characterized as a Goodwin growth cycle model (in the sense of Goodwin, 1967). The following features should make our contribution attractive to (at least) the Goodwin community and establish its raison d’être in the great stock of models in a Goodwinian tradition: while its reduced form is of a similarly clear structure, our model has a richer theoretical basis than Goodwin’s prototype model; the model is structurally stable, where the condition on the dynamic stability of the steady state growth path depends on the responsiveness of the animal spirits, or the herding effect; in the unstable case caused by strong herding the economy exhibits persistent and bounded cyclical behaviour in economic activity and income distribution, which prevails without the need to invoke an extrinsic nonlinearity of more or less flexible ceilings or floors.

The remainder of the paper is organized as follows. Section 2 puts forward the agents’ transition probabilities and derives the adjustment equation for the macroeconomic business climate from them. Specified in a so-called feedback index that determines the level of the transition probabilities, subsequently the concepts of self-reference and hetero-reference are introduced. Section 3 completes the model with a simple multiplier theory and a wage Phillips curve. It formulates the two differential equations, provides the local stability condition and then, which is the most interesting part, studies the global dynamics by means of phase diagrams and the Poincaré-Bendixson Theorem. To get a first feeling for the robustness of the dynamic properties, Section 4 examines two modifications. First a delay between desired and realized investment is discussed, which will add a third dimension to the system. The second modification concerns the specification of the herding mechanism, where in the feedback index the influence of the level of the business climate is replaced with its rate of change. After all, a verbal characterization of the model would hardly be affected by this alternative formulation. Section 5 concludes.

of as observing a random variable that follows a finite Markov process; Woodford, 1990). Two examples are Galí (1994) and Farmer and Guo (1994).
2 Dynamic adjustments of a general business climate

2.1 From microscopic transition probabilities to a macroscopic
differential equation

Let the business sector consist of $2N$ firms, whose number remains fixed over time. In each point in time a single firm can be characterized as being either optimistic or pessimistic about the future prospects of the economy. Designating an optimistic and pessimistic attitude by $(+)$ and $(-)$, respectively, let $n_t^+$, $n_t^-$ be the number of optimistic and pessimistic firms at time $t$ ($n_t^+ + n_t^- = 2N$). Next, put $n_t = (n_t^+ - n_t^-)/2$ and define $x_t = n_t/N$. Supposing that there is no systematic relationship between attitudes and the size of the firms, $x_t$ is the average attitude of firms or, as we will call it, the general business climate. Clearly, $-1 \leq x_t \leq 1$; optimism and pessimism balance in a state $x_t = 0$; and at $x_t > 0$ ($x_t < 0$) optimistic (pessimistic) firms form a majority.

Firms may well change their attitude over time. For the moment being, we consider this process in discrete time with adjustment periods of length $\Delta t > 0$. The changes will depend on a great variety of macroscopic aspects and idiosyncratic circumstances, which one will not want to specify in all of their details. It rather seems suitable to introduce random elements in this respect, in order to keep the modelling simple and to avoid arbitrary assumptions. Therefore, the basic concept to describe the changes in the business climate are the transition probabilities of the individual firms: at time $t$, let $\pi_t^{++}$ be the probability per unit of time that a firm changes from pessimistic to optimistic, and $\pi_t^{+-}$ the probability for an opposite change. More exactly, $\Delta t \pi_t^{++}$ is the probability that a firm who is pessimistic at $t$ will have become optimistic at the end of the period at $t + \Delta t$; and likewise $\Delta t \pi_t^{+-}$ for an optimistic firm. These probabilities are uniform across the population. They are, however, not fixed but vary with certain other macro variables in the economy (which will be discussed further below).

Let us beforehand examine how, given $\pi_t^{+-}$ and $\pi_t^{-+}$, the climate changes from $t$ to $t + \Delta t$.\footnote{The following argument draws on Alfarano and Lux (2005, Appendix A1 and A2).} If we consider the ‘excess’ number of optimistic firms $n_t$, it rises by 1 if a pessimistic firm becomes optimistic (when $n_t^+$ increases and $n_t^-$ decreases by 1). Symmetrically, $n_t$ declines by 1 if an optimistic firm turns pessimistic. Denoting by $k_t^+$ and $k_t^-$ the number of
converts of the first and second type, respectively, we have

\[ n_{t+\Delta t} = n_t + k_t^+ - k_t^- \]  

(1)

As the number of pessimistic firms at time \( t \) can be written as \( n_t^- = N - n_t \), the number \( k_t^+ \) of firms turning optimistic can be viewed as arising from \( N - n_t \) random draws of the type success/failure, each of which has probability \( \Delta t \pi_t^{+} \) for the event 'success', i.e., switch (and the complement for the no-switching event). The number of these events are then added up. Hence, the random variable \( k_t^+ \) has a binomial distribution \( B(N - n_t, \Delta t \pi_t^{+}). \)

Analogously, \( n_t^+ = N + n_t \) being the number of optimistic firms in \( t \), the random variable \( k_t^- \) is distributed as \( B(N + n_t, \Delta t \pi_t^{+}). \)

The expected values of these variables are \( E(k_t^+) = (N-n_t) \Delta t \pi_t^{+} \) and \( E(k_t^-) = (N+n_t) \Delta t \pi_t^{+} \), their variances amount to \( \text{Var}(k_t^+) = (N-n_t) \Delta t (1-\pi_t^{+}) \) and \( \text{Var}(k_t^-) = (N+n_t) \Delta t (1-\pi_t^{+}). \) If the expected values are large enough (exceeding 5 or 10), the two binomial distributions are (very) well approximated by the Gaussian distributions with the same first and second moments. Taking for granted that the population is large and \( n_t \) not too close to the boundaries \( \pm N \), we get

\[ k_t^+ = E(k_t^+) + \sqrt{\text{Var}(k_t^+)} \xi_t^+ , \quad k_t^- = E(k_t^-) + \sqrt{\text{Var}(k_t^-)} \xi_t^- \]  

(2)

where \( \xi_t^+ \) and \( \xi_t^- \) are two independent random draws from the standard normal distribution \( N(0,1) \) (with mean zero and variance equal to one). Furthermore, the difference between two normal distributions yields a normal distribution again. Its mean is the difference between the two single means, its variance the sum of the two single variances. For the random variable \( k_t = k_t^+ - k_t^- \), we thus have \( E(k_t) = \Delta t \cdot [(N-n_t) \pi_t^{+} - (N+n_t) \pi_t^{+}] = \Delta t \cdot [(1-x_t) \pi_t^{+} - (1+x_t) \pi_t^{+}] \cdot N \) and \( \text{Var}(k_t) = \Delta t \cdot [(1-x_t) \pi_t^{+} (1-\Delta t \pi_t^{+}) + (1+x_t) \pi_t^{+} (1-\Delta t \pi_t^{+})] \cdot N. \) It remains to divide (1) and (2) by \( N \) and we obtain,

\[ x_{t+\Delta t} = x_t + \Delta t \cdot [(1-x_t) \pi_t^{+} - (1+x_t) \pi_t^{+}] + \frac{\left[(\Delta t D_t)\sqrt{N}\right]}{\Delta t D_t} \xi_t \]  

\[ D_t := (1-x_t) \pi_t^{+} (1-\Delta t \pi_t^{+}) + (1+x_t) \pi_t^{+} (1-\Delta t \pi_t^{+}) , \quad \xi_t \sim N(0,1) \]  

(3)

Equation (3) abstracts from the many individual and accidental switches in the firms’ attitudes and summarizes them in a macroscopic stochastic equation that governs the changes

\footnote{A binomial distribution \( B(N, \pi) \) is the probability distribution for the number of successes \( k \) in a sequence of \( n \) independent sucess/failure experiments, each of which yields success with probability \( \pi \). The probability of getting exactly \( k \) successes is given by \( \binom{N}{k} \pi^k (1-\pi)^{N-k} \), the mean is \( N\pi \), and the variance \( N\pi(1-\pi) \).}
in the climate. In statistical mechanics this type of equation is known as the Langevin equation. The equation is there usually derived by first setting up the entire probability distribution $P = P(x, z, t)$ of $x$ and a possible set of additional variables $z$, and then analyzing its change over time. For this purpose the powerful tool of the Fokker-Planck equation (FPE) is employed, which is itself a second-order approximation. There is indeed an intimate connection between FPE and the Langevin equation. Regarding (3) it is shown by the fact that the term in the first square brackets is the drift coefficient in FPE, and $D_t$ corresponds to its fluctuation or diffusion term.\footnote{See Weidlich and Haag (1983, pp. 22 – 26) for a succinct presentation of the relationship between FPE and the Langevin equation in continuous time. An example of this treatment in discrete time is Alfarano et al. (2005, pp. 23f, 46f).}

For our present purpose we can stick with (3). It suffices to note that the noise level in the equation decreases with the size of the population. Then, for the rest of paper, let $N$ be so large that this intrinsic stochastic component can be safely ignored. For the theoretical analysis it is furthermore convenient to let the adjustment period shrink to zero, $\Delta t \to 0$. Equation (3) remains well-defined and, dropping the time index, the business climate is seen to be governed by a differential equation,

$$\dot{x} = (1 - x) \pi^{-+} - (1 + x) \pi^{+-}$$

(4)

Clearly, this equation can be taken on its own and treated like any other ordinary differential equation. The focus therefore shifts to the specification of the transition probabilities $\pi^{-+}$ and $\pi^{+-}$.

### 2.2 Feedbacks in the individual transition probabilities

The transition probabilities will change in response to the variations of a set of several variables that the firms observe. These effects can generally be summarized in a single feedback index $f$, which attains positive and negative values in different stages the economy goes through. Let positive and negative be related to the probability $\pi^{-+}$ of switching from pessimistic to optimistic, that is, an increase in the feedback index increases $\pi^{-+}$ and decreases the complementary probability $\pi^{+-}$.

It is an obvious concept, which Weidlich and Haag (1983) have also found very helpful in their formal analysis, to assume that the changes of the transition probabilities depend...
on the changes of the index $f$ in a linear way. More precisely, on the relative changes, so that we have $d\pi^-/\pi^- = \alpha df$ for some constant $\alpha$. As it is only a matter of scale of the feedback index, $\alpha$ can be taken to be unity without loss of generality. Symmetry is another natural assumption to make, which gives us $d\pi^+/\pi^+ = -df$. Introducing $\nu > 0$ as an ‘integration constant’, the specification of the transition probabilities reads (‘exp’ being the exponential function),

$$
\pi^- = \pi^-(f) = \nu \exp(f), \quad \pi^+ = \pi^+(f) = \nu \exp(-f)
$$

Certainly, (5) ensures positive values of the probabilities. The complementary condition that the feedback index is bounded, such that the probabilities are less than unity, should be a property of the model into which (5) is incorporated, (or the outcome of an empirical estimation). The adjustment equation (4) of the business climate $x$ itself becomes

$$
\dot{x} = \nu [(1-x)\exp(f) - (1+x)\exp(-f)]
$$

An individual firm may well have its specific reasons to maintain or switch to one of the attitudes. With more information on such a firm its behaviour could then be described in terms of conditional probabilities, or each firm could be equipped with transition probabilities of its own. Intuition says that these firm-specific probabilities (or rather feedback indices) would average out across the whole population.\(^7\) In any case, from our macroscopic point of view we only need to refer to the unconditional probabilities, which are uniformly, or representatively, given by (5).

A special feature of the specification of the transition probabilities is $\pi^- = \pi^+ = \nu > 0$ when $f = 0$. Hence even in the absence of active feedback forces or when the different feedback variables neutralize each other, the individual firms will still change their attitude with a positive probability. While these reversals, which can occur in either direction, are to be ascribed to idiosyncratic circumstances, from our bird’s-eye view they appear as purely random and, given that the population is large, will cancel out if we consider them in the aggregate, implying $\dot{x} = 0$ in (6). Since $\nu$ in (5) measures the general responsiveness (per unit of time) of the transition probabilities to the arrival of new information, it is not just a

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\(^7\)Under some mild conditions it should be possible to employ the Central Limit Theorem and prove this mathematically. Simone Alfarano made some exercises in a related framework to spell out the details (private communication).
technical integration coefficient but can be characterized as a flexibility parameter (Weidlich and Haag, 1983, p. 41).

As indicated in the Introduction, in specifying the feedback two concepts are considered. The first is the concept of self-reference, which grasps the idea that waves of optimistic and pessimistic sentiment are generated by means of a self-exciting mechanism. Accordingly, the probability of switching from pessimistic to optimistic is larger than in the opposite direction if the prevailing disposition in the population of firms is already optimistic (and vice versa). Setting up the index $f$ in an additive way, the first term in $f$ is therefore given by the current business climate $x$, multiplied by a constant coefficient $\phi_x$ that measures how strongly firms tend to join the current majority. Borrowing the expression from the financial markets literature, $\phi_x$ can be referred to as the model’s herding parameter.\(^8\)

While the first concept essentially constitutes a process of herding, as it has just been called, we additionally introduce a hetero-referential mechanism. It takes account of the more “objective” factors that may induce firms to change their attitude. These factors may reinforce or put a curb on the herding dynamics, or they may even reverse it. In a Goodwinian spirit and regarding the prospects of their investment, we assume that higher profits in the economy increase the probability of a pessimistic firm to become optimistic or, if it is already optimistic, to maintain this attitude. This overall profitability is measured by the share of aggregate profits in national income. Thus the profit share can feature as the second term in the feedback index: entering it in a positive way, the transition probability $\pi^{-+}$ increases when the profit share increases.

Instead of the profit share, it will be more convenient in the presentation of the model to refer to its complement, the wage share, which we denote by $v$. This macro variable is included as a negative entry in the feedback index, such that ceteris paribus a value of $v$ below (above) a benchmark $v_o$ induces a higher (lower) probability $\pi^{-+}$. The strength of this objective feedback factor in relation to the business climate is represented by a positive

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\(^8\)The firms might also have a general predisposition towards optimism or pessimism, which could be easily captured by including a positive or negative constant in the feedback index $f$. The mechanisms of the model would not be affected in any way by this non-neutrality, only the steady state growth rate would be somewhat ‘distorted’. 
coefficient \( \phi_v \). In sum, the feedback index \( f \) in (6) is specified as

\[
f = \phi_x x - \phi_v (v - v^o)
\]  

Equations (6) and (7) show that the macroeconomic business climate is driven by itself (the auto-feedback) and by the macroeconomic wage share (the hetero-feedback). Note that in a state of balanced income distribution and balanced optimism and pessimism, when \( v = v^o \) and \( x = 0 \), the auto-feedback is positive and thus destabilizing if \( \phi_x > 1 \), and it is negative and thus stabilizing if \( \phi_x < 1 \). This follows easily from partially differentiating \( \dot{x} \) with respect to \( x \), which evaluated at \( x = 0 \) yields \( \partial \dot{x} / \partial x = 2 \nu (\phi_x - 1) \). The herding parameter \( \phi_x \) will also be of central importance in the full macro model that we are going to formulate in the next section.

Before, however, we should return to eq. (6) and make an important observation whose economic significance goes far beyond the present modelling framework. Independently of the way in which the feedback index may be specified, if \( f \) remains bounded in a model, the boundaries \( \pm 1 \) of the climate \( x \) are repelling. Clearly, as \( x \) gets close to 1 in (6), the first term in square brackets tends to vanish. Since the second term is positive and bounded away from zero, the time derivative \( \dot{x} \) will eventually turn negative and the climate will decrease. Conversely, if \( x \) gets sufficiently close to \(-1 \), the derivative will be positive and \( x \) will rise again. We emphasize that apart from the exponential function in the specification (5) of the transition probabilities, which there quite innocently was only introduced to make the relative changes of \( \pi^{-+} \) and \( \pi^{+-} \) linearly dependent on the changes in \( f \) and, as a consequence, to prevent the probabilities from falling down to zero, no further nonlinearity is required for this global self-stabilization mechanism to work out. As far as the adjustments of the aggregate climate variable is concerned, modelling could be done without designing any extrinsic ceiling or floor.

3 A Goodwinian prototype model

3.1 Utilization and changes in income distribution

To build up a small macrodynamic model on the basis of the herding dynamics specified above, we only have to combine the differential equation (6) for the business climate with an equation that determines the motions of the external norm \( v \) in the feedback index (7),
which is here the wage share. Two elements are sufficient for that. First, as in many other models of Goodwinian type to discuss income distribution dynamics, the motions of the wage share can be easily derived from a wage Phillips curve with economic activity as the driving force. Second, following the Keynesian tradition and viewing economic activity as basically demand-driven, it is convenient to postulate a direct influence of the business climate on investment and thus, via the multiplier, on aggregate demand and finally output. We are now going to formulate these ideas in a simple way.

To begin with the demand side, we concentrate on fixed investment as the only active component of demand and conceive it as an increasing function of the business climate. Considering the micro level, each individual firm has two investment options, which are given by a minimal growth rate $g_{\text{min}}$ if the firm is pessimistic, and a maximal growth rate $g_{\text{max}}$ if the firm is optimistic. Let $g^o$ be the growth rate of the aggregate capital stock which will prevail in a balanced state of the business climate, when $x = 0$. Since the aggregate capital growth rate $g$ equals $g_{\text{min}}$ if $x = -1$, and $g_{\text{max}}$ if $x = 1$, the equilibrium growth rate is

$$g^o = \left( g_{\text{min}} + g_{\text{max}} \right) / 2 \quad (8)$$

Generally, a climate $x$ gives rise to the capital growth rate

$$g = g(x) = g^o + \beta_{gx} x \quad (\beta_{gx} := (g_{\text{max}} - g_{\text{min}}) / 2) \quad (9)$$

The other components of demand are assumed to be proportional either to the capital stock $K$, which may serve as a trend indicator, or to disposable income, which itself is supposed to be a linear function of total income $Y$ and of $K$. We furthermore assume a closed economy and continuous temporary equilibrium on the goods markets, so $Y$ is also total output. With suitable constants $c_1, \ldots, c_4$ and $s$ the economy’s (constant) propensity to save out of disposable income, the market clearing condition gives $Y = \text{net investment} + \text{capital depreciation} + \text{consumption} + \text{government spending} = gK + c_1K + (1-s) \cdot \text{disposable income} + c_2K = gK + (c_1 + c_2)K + (1-s)(Y + c_3K) = gK + c_4K + (1-s)Y$. Put $\beta_u = c_4/s$ and let $u = Y/K$ denote the output-capital ratio, which indicates the utilization of the capital stock. Dividing the market clearing condition by $K$ and solving it for $u$, a simple
multiplier relationship is obtained,\textsuperscript{9}

\[ u = g(x)/s + \beta_u = (g^o + \beta g x)/s + \beta_u \]  

\text{(10)}

Obviously, the variations of capital utilization, which represent the stages of the business cycle, can be directly identified with the variations of the business climate.

To model the variations of the wage share, let \( w \) be the nominal wage rate, \( z \) labour productivity, \( p \) the price level, and \( u^o \) a ‘normal’ rate of capital utilization. The Phillips curve mentioned above can then be set up as a money wage Phillips curve, which reads,

\[ \dot{w} = \hat{z} + \hat{p} + \beta_w (1-v) (u-u^o) \]  

\text{(11)}

The structural coefficients behind \( \beta_u \) in (10) are assumed to be consistent with \( u^o \), that is, \( u^o = g(0)/s + \beta_u = g^o/s + \beta_u \).

The term \( 1-v \) is a convenient device that below will prevent wages from totally exhausting the national product. The responsiveness of money wages to the changes in economic activity are thus measured by the product \( \beta_w (1-v) \), whose variations will be negligible if the amplitude of the wage share is small.

The sum of productivity growth \( \hat{z} \) and price inflation \( \hat{p} \) in (11) constitutes a reference level for wage inflation, in the sense that in a state of normal utilization \( u = u^o \), the wage share \( v = wL/pY = (w/p)/z \) remains invariant (\( L \) being employment), which it does by virtue of \( \dot{v} = \dot{w} - \dot{p} - \dot{z} = 0 \). Current wage inflation exceeds reference wage inflation if the capital stock is overutilized, \( u > u^o \), a situation for which one may assume that it goes along with an unemployment rate below its ‘natural’ level; conversely, wage inflation falls short of \( \hat{z} + \hat{p} \) in a state of underutilization.

A differential equation for the wage share is now readily obtained. Logarithmic differentiation of \( v \) yields \( \dot{v} = \dot{w} - \hat{p} - \hat{z} = \beta_w (u-u^o) (1-v) \). Plugging in utilization from (10),

\text{\textsuperscript{9}After solving for \( u \), the structure of eq. (10) would be preserved if utilization is added as another determinant of investment in (9), such that \( g = g^o + \beta_g x + \text{const} \cdot (u-u^o) \), where \( u^o \) is the level of normal utilization introduced in a moment.}

\text{\textsuperscript{10}On the basis of the steady state calibration in Chiarella et al. (2005, pp. 85f), we could set \( s = 0.069 \) and \( \beta_u = 0.266 \). In this way an annualized steady state growth rate \( g^o = 3\% \) yields an equilibrium output-capital ratio \( u^o = 0.70 \) (for likewise annualized production flows), which it is argued at the same place is a reasonable order of magnitude. However, even if relatively wide variations of \( u \) were permitted, the associated deviations of the capital growth rates from \( g^o \) would still be unrealistically low. Or one accepts that a higher savings propensity to mitigate this problem implies unrealistic macroeconomic proportions elsewhere in the model. This is the price one has to pay for the simple specification of the demand side with its fixed proportions, as well as for the convenience of temporary equilibrium on the goods market.}
taking the consistency condition on $\beta_u$ into account, and putting $\beta_{gs} = \beta_{gx}/s$, we have

$$\dot{v} = \beta_w \beta_{gs} v (1 - v) x$$

(12)

To sum up, equations (6) and (12), where (7) has to be substituted for $f$ in (6), constitute a system of two differential equations in the state of confidence, $x$, and the wage share, $v$.

### 3.2 Local and global analysis of the complete model

For a mathematical analysis of the global dynamics it is useful to work with hyperbolic functions in the climate adjustments. The definitions for the hyperbolic sine and cosine (sinh and cosh) allow us to reformulate (6) as

$$\dot{x} = 2\nu \left\{ \frac{\exp(f) - \exp(-f)}{2} - x \left[ \exp(f) + \exp(-f) \right]/2 \right\} = 2\nu \left[ \sinh(f) - x \cosh(f) \right].$$

With $\tanh = \sinh / \cosh$ for the hyperbolic tangent and writing out the feedback index in full, this equation becomes

$$\dot{x} = 2\nu \left\{ \tanh[\phi_x x - \phi_v (v - v^o)] - x \right\} \cosh[\phi_x x - \phi_v (v - v^o)]$$

(13)

The system to be investigated is thus given by (12) and (13).\footnote{Besides the few remarks in the text, only the following properties of the hyperbolic functions will be used in the analysis of (13): $\tanh(0) = 0$; $\cosh(0) = 1$; $\tanh'(0) = 1/[\cosh(0)]^2$.}

Clearly, $v = v^o$ and $x = 0$ constitute a point of rest. Since $\dot{v} = 0$ implies $x = 0$ (if we disregard the meaningless cases $v = 0$ and $v = 1$), cosh is everywhere positive, and tanh is a strictly increasing function, this equilibrium is unique. It goes without saying that aggregate output and capital grow in line in this situation, at the rate $g^o$. The individual firms are, however, heterogeneous in this respect: half of them have their capital stock growing at the rate $g_{min}$, and the other half at the rate $g_{max}$. In addition, there is some change going on under the macroeconomic surface, since, as observed before, individual firms will still randomly switch from one attitude, or growth rate, to the other.

Regarding a local stability analysis of (12) and (13), the Jacobian matrix $J$ that is associated with its equilibrium point is easily computed as

$$J = \begin{bmatrix} 0 & \beta_w \beta_{gs} v^o (1 - v^o) \\ -2\nu \phi_v & 2\nu (\phi_x - 1) \end{bmatrix}$$

(14)

The determinant is unambiguously positive, and the sign of trace $J = 2\nu (\phi_x - 1)$ depends on the coefficient $\phi_x$ alone. The equilibrium is therefore established to be locally asymptotically
stable if the herding parameter $\phi_x$ is less than unity, whereas it is locally repelling if $\phi_x$ is greater than one.

The global properties of (12) and (13) can be studied by means of phase diagrams. The isocline $\dot{v} = 0$ is trivially given by the straight line $x = 0$, and fortunately also the isocline of the business climate, $\dot{x} = 0$, can be explicitly computed. To this end note that $\dot{x} = 0$ if (and only if) the term in curly brackets in (13) vanishes. Applying the inverse function $\text{arctanh}(\cdot)$ to both sides of the equation $\tanh[\phi_x x - \phi_v (v - v^o)] = x$, using the identity $\text{arctanh}(x) = (1/2) \cdot \ln \left( \frac{1 + x}{1 - x} \right)$, and solving the resulting equation for $v$, we have that $\dot{x} = 0$ is equivalent to

$$v = v_{iso}(x) := v^o + \frac{1}{\phi_v} \left[ \phi_x x - \frac{1}{2} \ln \left( \frac{1 + x}{1 - x} \right) \right]$$  \hspace{1cm} (15)$$

$v_{iso}(x)$ tends to $-\infty$ as $x$ approaches 1, and to $+\infty$ as the climate approaches $-1$ from the right. The derivative of the function is

$$dv_{iso}(x)/dx = \left[ \phi_x - \frac{1}{1 - x^2} \right] / \phi_v$$  \hspace{1cm} (16)$$

Since the fraction in (16) is always equal to or greater than one, it follows that in the stable case, when $\phi_x < 1$, the isocline is everywhere downward sloping. Figure 1a shows the phase diagram thus resulting. As it drawn, the diagram suggests that the equilibrium is approached in a cyclical manner. This is ensured if $\phi_x$ is sufficiently close to 1. The reason is that with the positive determinant and trace $J = 2 \nu (\phi_x - 1)$, the Jacobian has two purely imaginary eigen-values if $\phi_x = 1$, and the non-degenerate imaginary parts of the eigen-values are maintained if $\phi_x$ does not depart too much from unity. However, as $\phi_x$ decreases further, the slope of $v_{iso}(\cdot)$ becomes steeper and so convergence toward the equilibrium may eventually occur in a monotonic way, from the north-west and south-east, respectively.

Figure 1a furthermore suggests that stability is not only local but also global, which was confirmed in all our numerical explorations. Nevertheless, it cannot generally be ruled out that trajectories starting in a far distance from the equilibrium do not converge to it any longer. Should this happen, the arguments of the unstable case apply and these trajectories converge toward a closed orbit, quite similar to what is shown in the outer regions of Figure 1b.

If the parameter $\phi_x$ exceeds unity, the slope (16) is positive at $x = 0$. Since it is negative for $x$ close to $\pm 1$, there must be at least one local minimum between $-1$ and 0,
and one local maximum between 0 and 1. By equating (16) to zero it is easily found that these are the only ones, the two solutions being $x_{1,2} = \pm \sqrt{(\phi_x - 1)/\phi_x}$. Thus, the phase diagram in Figure 1b is obtained.

It has already been noted that $\pm 1$ are reflecting boundaries for the climate $x$. As can be seen from (12), the wage $v$ cannot reach its lower and upper boundaries 0 and 1, either. Hence, all meaningful motions of $v$ and $x$ are contained within in a compact set, which is the basic condition for the Poincaré-Bendixson Theorem to apply. Given that the equilibrium is unique and locally repelling, each trajectory must accordingly converge toward a closed orbit.

It might be felt that, as illustrated in Figure 1b, there is only one such periodic motion. We know that the trajectories are spiralling outward when being close to the equilibrium, and they are spiralling inward in the outer regions of the phase plane. If one had an index that measures the force at which the system spirals inward at positive values, and the force at which it spirals outward at negative values, then, according to intuition, this index should steadily increase as one goes from within to the outside. If the index changes continuously, such that it is zero on a closed orbit, the existence of just one periodic orbit would be established; it would then be a unique limit cycle indeed. On the other hand, the monotonicity property is plausible. After all, the model exhibits only one essential nonlinearity, in the exponential functions in eqs (6) or (13), respectively, and owing to their

Figure 1: Phase diagrams of system (12) and (13).
convexity, this nonlinearity might be said to be monotonic. Generally, however, it seems hard to find a rigorous mathematical proof for a unique limit cycle.

Since the climate index is directly connected to the rate of utilization in (10), the motions of \( x \) can also be identified with the motions of the macroeconomic employment rate. The cyclical trajectories in Figure 1 are therefore in close correspondence to the closed orbits of the wage share and the employment rate in the prototype Goodwin model (supposing that employment and output move quite in line). The present model, however, has a richer theoretical basis and even a microeconomic underpinning. In addition, it is structurally stable. This means, first, that the steady state position can be asymptotically stable or unstable, where the condition determining this property has a clear economic message: strong herding tends to be destabilizing. A second point is that there does not exist a continuum of closed orbits; our periodic trajectories are at least locally unique.

4 Respecifications of the model

The task of a prototype model is to cast ideas about a few elementary feedback mechanisms into a simple form that is easy to analyze and whose results make good economic sense. Having this achieved in the previous section, a next question is for the robustness of the properties of the model when some of its details are slightly modified (a study of their generalizations would then be a further step). In this respect, we are here concerned with two topics. First, we will introduce a lag into the channel from the business climate to aggregate demand. Second, we maintain the idea of the auto-feedback in the climate adjustments, but introduce an accelerationist element into the specification of the feedback index.

4.1 Delays in investment demand

In the basic model it has been assumed that a change in the climate index instantaneously translates into investment demand. In this section we account for the fact that it will typically take some time until the change in a manager’s attitude will find its way into an approved decision within the firm, an order to one or several other firms, the production of the corresponding goods and, at the end, the delivery of the final product. Especially the
third lag will be considerable for many items of fixed investment, the so-called gestation period.

A convenient way to model the delays just sketched is the distinction between a desired and an actual capital growth rate, $g^d$ and $g$, respectively, and the assumption that $g$ partially adjusts toward $g^d$. If $\beta_{gg} > 0$ denotes a constant speed of adjustment, this gives us $\dot{g} = \beta_{gg} (g^d - g)$. Desired capital growth continues to be given by eq. (9), now written as $g^d = g^d(x) = g^o + \beta_{gx} x$. Taken together, the differential equation for $g$ as a third dynamic state variable reads,

$$\dot{g} = \beta_{gg} (g^o + \beta_{gx} x - g)$$

(17)

Instead of (10), capital utilization is now given by $u = g/s + \beta_u$. Normal utilization being $u^o = g^o/s + \beta_u$, the “output gap” in the Phillips curve becomes $u - u^o = (g - g^o)/s$, so that the climate variable in (12) for the changes of the wage share is replaced with this gap in the capital growth rates. The adjustments of the climate index are not directly affected by the delays in effective demand. Thus (17) is combined with the two differential equations,

$$\dot{v} = (\beta_w/s) v (1 - v) (g - g^o)$$

(18)

$$\dot{x} = 2\nu \{ \tanh[\phi_x x - \phi_v (v - v^o)] - x \} \cosh[\phi_x x - \phi_v (v - v^o)]$$

(19)

The three-dimensional system (17)–(19) has a unique equilibrium, $(g, v, x) = (g^o, v^o, 0)$. The Jacobian evaluated at this point is

$$J = \begin{bmatrix}
\beta_{gg} & 0 & \beta_{gg} \beta_{gx} \\
(\beta_w/s) v^o (1 - v^o) & 0 & 0 \\
0 & -2\nu \phi_v & 2\nu (\phi_x - 1)
\end{bmatrix}$$

The many zero entries allow an easy computation of the Routh-Hurwitz stability conditions. The necessary and sufficient conditions for local asymptotic stability, i.e., that all eigenvalues have negative real parts, are $A_1 > 0$, $A_2 > 0$, $A_3 > 0$, $B > 0$, where:

$$A_1 = -\text{trace} J = \beta_{gg} + 2\nu (1 - \phi_x)$$

$$A_2 = J_1 + J_2 + J_3 = 2\nu \beta_{gg} (1 - \phi_x)$$

$$A_3 = -\text{det} J = 2\nu \phi_v \beta_{gg} \beta_{gx} \beta_w / s$$

$$B = A_1 A_2 - A_3 = 2\nu \beta_{gg} \{ (1 - \phi_x) [\beta_{gg} + 2\nu (1 - \phi_x)] - \phi_v \beta_{gx} \beta_w / s \}$$

($J_i$ are the three principal minors, the determinants of the $2 \times 2$ submatrices obtained after deleting the $i$-th row and column of $J$). The equilibrium of this economy continues to
be unstable if the herding parameter exceeds one, $\phi_x > 1$. However, unity constitutes no longer the unequivocal benchmark for stability, via the mixed term $B$ the other parameters take some effect, too. At small values of $\beta_{gg}$ and moderate values of $\phi_v, \beta_{gx}, \beta_w/s$, the coefficient $\phi_x$ must be sufficiently below one for $B$ to be positive; that is, herding must be sufficiently weak. In particular, it is perhaps a bit surprising that strong feedback reactions $\phi_v$ of the firms to reductions of their profitability tend to be destabilizing. Note also the destabilizing influence of wage flexibility, i.e., the slope parameter $\beta_w$ in the Phillips curve. On the other hand, in the presence of $\phi_x < 1$ high enough adjustment speeds $\beta_{gg}$ in (17) could always stabilize the equilibrium. This is as it should be, since in the limit, $\beta_{gg} \to \infty$, the prototype model would be re-established.

Of course, a global analysis of (17) – (19) with the nice and clear-cut results of the prototype model is here not possible. We can nevertheless make two observations. First, since the determinant of $J$ is always negative, loss of stability as one of the parameters varies occurs by way of a Hopf bifurcation, which at least in a vicinity of the critical parameter set gives rise to a pair of complex eigen-values and thus oscillatory motions. Second, all trajectories remain within a compact set. This has already been noted for the wage share and the climate index. In addition, it is easily seen in (17) that the capital growth rate $g$ cannot leave the interval $[g^o - \beta_{gx}; g^o + \beta_{gx}]$ (since $\pm 1$ are the lower and upper bounds of $x$, which it actually will never hit). It follows from this boundedness (and the uniqueness of the equilibrium) that monotonic divergent movements of the trajectories cannot be sustained; sooner or later they must bend around and display some kind of cyclical behaviour. Generally, however, we cannot make similarly elementary statements about the regularity of these fluctuating motions; in particular, whether for all three variables the upper and lower turning points of the times series will be above and below the equilibrium values. On the basis of the experience with the prototype model and the fact that the present version of the model is not so much different, this is a credible working hypothesis—which would then have to be checked in a careful numerical analysis.

4.2 The moving-flock effect

Even without economic theory, financial markets know the notion of herding and in this connection occasionally indulge in allusions to market sheep. If we take up the metaphor
of a flock always in search for a greener grass, then the individual sheep may not wait until the majority has gathered on a better spot and bleats invitingly, it rather follows the other sheep in its neighbourhood as soon as they begin moving. In terms of our feedback index $f$, this means that firms may not only be influenced by the level of the current majority of optimism or pessimism, but they may also respond to the motions of this crowd, i.e., to the recent changes in the general climate. Accordingly, in addition to $x$ the feedback index may also include the rate of change of $x$. For easy reference, let us call the impact of the level of $x$ on the feedback index the majority effect, and the influence of the changes in $x$ the moving-flock effect.

Both concepts are well described as herding. A priori it is not obvious why one specification should be rejected in favour of the other. In principle, this should be an empirical question. In another study (Franke, 2007) we have tested the present approach to model a general business climate and its changes over time with two leading sentiment indicators for the German economy (the ZEW and ifo indices for six-months ahead expectations of the general business situation), where instead of the wage share the hetero-referential variable in the feedback index was a measure of economic activity. The estimations were quite successful, and for both indices the moving-flock effect turned out highly significant. It was even justifiable, and for reasons of parsimony preferable, to discard the majority effect altogether. Against this background it should be examined what it means for our small macro model if the majority effect is replaced with the moving-flock effect.

While the empirical estimations were carried out in discrete time and the rates of change were of the form $(x_t - x_{t-\tau})/\tau$ (where the exact lags $\tau$ are important for the goodness-of-fit), we have to work with time derivatives if, for better comparability, we want to remain in the previous continuous-time setting. Introducing a positive coefficient $\phi_{dx}$ on $\dot{x}$, the feedback index $f$ in (7) changes to $f = \phi_{dx}\dot{x} - \phi_{v}(v - v_o)$. Conceptually, $\dot{x}$ is here a backward-derivative, $\dot{x}(t) = \lim[x(t) - x(t-\Delta t)]/\Delta t$ for $\Delta t \to 0$, whereas in the differential equation for $x$ the expression on the left-hand side is a forward-derivative, $\dot{x}(t) = \lim[x(t+\Delta t) - x(t)]/\Delta t$. In the mathematical treatment, however, there is no longer any role for these distinctions. Returning to the direct impact of the climate on investment and capital utilization in Section 3 (or $\beta_{gg} = \infty$ when we refer to Section 4.1), we again use (12) for the motions of the wage share and so get the following dynamic system

\[
\dot{v} = \beta_w \beta_{gs} v (1 - v) x
\]  

(20)
\begin{align*}
\dot{x} &= h(v, x, \dot{x}) \\
&:= 2\nu \left\{ \tanh[\phi_{dx}\dot{x} - \phi_v(v-v^o)] - x \right\} \cosh[\phi_{dx}\dot{x} - \phi_v(v-v^o)] \tag{21}
\end{align*}

Technically, the adjustments of the climate are now described by an implicit differential equation for \(\dot{x}\). For numerical simulations it would, of course, be appropriate to discretize (21) along the lines sketched above, so that we get a recursive system and no problems arise.

For a mathematical analysis, eq. (21) has to be solved for the derivative \(\dot{x}\). An explicit solution will (probably) not be possible, but the Implicit Function Theorem ensures the existence of such a solution, which we write as

\begin{equation}
\dot{x} = H(v, x) \tag{22}
\end{equation}

The properties of the (continuously differentiable) function \(H(\cdot, \cdot)\) can be studied by computing its partial derivatives.

More precisely, a unique function \(H(\cdot, \cdot)\) exists (with finite values) in a neighbourhood of the equilibrium \((v, x) = (v^o, 0)\). There might be points \((\tilde{v}, \tilde{x})\) for which the partial derivative \(\partial h[v, x, H(v, x)]/\partial \dot{x}\) converges to 1 as \((v, x) \to (\tilde{v}, \tilde{x})\). Here \(H(\tilde{v}, \tilde{x})\) could exist and would not be differentiable at this point, or \(H(v, x)\) tends to plus or minus infinity as \((v, x)\) tends to \((\tilde{v}, \tilde{x})\). A solution path of the climate, \(x = x(t)\), as a continuous function of time will, however, still exist, only that it will have a kink at the time \(t_k\) when \(x(t_k) = \tilde{x}\).

As concerns the derivatives of the function \(H(\cdot, \cdot)\), they are easily obtained by applying the Implicit Function Theorem to the identity \(h(v, x, \dot{x}) - \dot{x} = 0\). This yields

\begin{equation}
\frac{\partial H(v, x)}{\partial z} = \frac{-\partial h[v, x, H(v, x)]/\partial z}{\partial h[v, x, H(v, x)]/\partial \dot{x} - 1} \quad (z = v, x) \tag{23}
\end{equation}

The derivatives are less cumbersome than they might look at first sight, and they are particularly simple when they are evaluated at the equilibrium point. The Jacobian of (20) and (22) then reads,

\begin{equation}
J = \begin{bmatrix}
0 & \beta_w \beta_g s v^o (1 - v^o) \\
\frac{2\nu \phi_v}{2\nu \phi_{dx} - 1} & \frac{2\nu}{2\nu \phi_{dx} - 1}
\end{bmatrix} \tag{24}
\end{equation}

With respect to stability it will be expected that the herding coefficient \(\phi_{dx}\) takes the role of \(\phi_x\) in Section 3. In contrast to the Jacobian (14), however, where \(\phi_x\) only enters in one of the diagonal elements, \(\phi_{dx}\) in (24) also shows up in the corresponding off-diagonal element. This implies that the sign of the determinant of \(J\) is no longer unique, as
\[ \det J = -2\nu \phi_v \beta_u \beta_g s \nu^\phi (1-\nu^\phi)/(2\nu \phi_{dv} - 1). \] Since the sign of the trace of \( J \) equals the sign of \((2\nu \phi_{dx} - 1)\), it can be concluded that the equilibrium of system (20), (21) is locally asymptotically stable if \( \phi_{dx} < 1/2\nu \). The stability condition can thus be expressed in the same words as in Section 3.2: local asymptotic stability prevails if the herding effect is sufficiently weak.

Conversely, a strong herding effect \((\phi_{dx} > 1/2\nu)\) will destabilize the equilibrium. The difference from Section 3.2 is that here instability tends to be more pronounced since, owing to \( \det J < 0 \) in this case, the equilibrium is of the saddle-point type. As a consequence, local divergence occurs in a monotonic way.

Nevertheless, although there is locally no scope for cyclical motions, the monotonic behavior cannot be sustained in the outer regions of the state space. The global self-stabilizing mechanism in the adjustments of the business climate that has been pointed out in the discussion of eq. (13) is equally at work in (21). Hence system (20), (21), too, exhibits persistent fluctuations in the wage share \( v \) and the climate index \( x \).

It may be noted that for a more rigorous mathematical analysis the Poincaré-Bendixson Theorem is no longer available, simply because the function \( H(\cdot, \cdot) \) in (22) might not be continuously differentiable. Owing to the possible discontinuities or even poles of \( H(\cdot, \cdot) \), it may also be problematic to derive a phase diagram. These problems should not be too surprising if we go back to the discrete-time foundations of the climate adjustment equation with a finite (though arbitrarily small) period \( \Delta t \). In this setting the abovementioned distinction between forward and backward time derivatives would be regained and we obtain a three-dimensional discrete-time system (with state variable \( v_t, x_t, x_{t-\Delta t} \)). Locally there may be a perfect correspondence between a 2D continuous-time and a 3D discrete-time system, but one cannot count on this to hold also globally on the entire \((v, x)\)-plane.

While we have emphasized the self-stabilizing forces in eqs (13) and (21) if the equilibrium is unstable, it should be added that, in the present moving-flock variant of our model, they are are relatively weak. A few exploratory simulations with the discrete-time system are sufficient to show that even with an extremely short adjustment period \( \Delta t \), the climate comes very close to its boundaries \( \pm 1 \). These motions are moreover so fast that in comparison the oscillations in the wage share are of secondary importance. That is, the accelerationist element in the climate adjustments is so strong that the dynamics is essentially reduced to extreme up and downs of the climate variable itself. If one adheres to
the paradigm of an unstable equilibrium and self-sustaining cyclical behaviour around it, then system (20), (21) or its discrete-time counterpart does not seem to be a very attractive model.

Despite the model’s unpleasant properties, the moving-flock specification of herding should not be dismissed too early. First, one cannot simply ignore empirical evidence of this mechanism, as it was worked out by Franke (2007) for German survey expectations. Second, the influence of the (backward) time derivative in the feedback index is probably too drastic a specification. The strong accelerationist tendencies should weaken if the moving-flock element is based on longer lags in the rates of change of $x$ (in a stochastic framework such longer lags would be mandatory, anyway). Third, generally herding may be represented by a combination of the moving-flock effect and the majority effect. Fourth, the accelerationist tendencies may also be less dominating if the rest of the model has a richer feedback structure and the business climate acts on it in a less direct and less absolute way.

5 Conclusion

The paper has advanced a convenient modelling strategy to build up macroeconomic adjustments from the micro level of individual firms or agents having two modes of behaviour. As a very attractive feature it was, in particular, pointed out that the resulting dynamic macro equation contains a mechanism of global self-stabilization with reflecting boundaries for the aggregate index variable. It is obvious that this approach can be applied to a great variety of decision problems in economics. Here we focussed on an interpretation of the index variable as a general business climate determining fixed investment, which can give the Keynesian notion of animal spirits a more precise and tractable meaning.

It was moreover shown that the climate and its adjustments can be easily combined with other elements from the (heterodox) macroeconomic literature. Invoking a simple multiplier and an ordinary wage Phillips curve, we actually obtained a parsimonious version of an enriched demand-driven Goodwin growth cycle model. In contrast to Goodwin’s design, however, it admits asymptotic stability as well as instability of the steady state, responsible for which is exclusively the intensity of herding in the formation of the business climate. Most nicely, if herding is strong such that the equilibrium is repelling, the economy will necessarily converge toward a trajectory with persistent and periodic oscillations.
in the two Goodwin variables economic activity and income distribution. To check the robustness of the results, we also briefly checked two modifications and demonstrated that they qualitatively survive.

Future research may be concerned with integrating the business climate adjustments into other and more encompassing macro models, which also have a role for monetary (and perhaps other) policy. A second topic is an extension of the microeconomic framework to three states, so that agents do not only have the choice between two extremes like optimism and pessimism but are also allowed to switch to an intermediate option. First explorations indicate that even with strong herding two regimes may prevail: one exhibiting persistent and bounded fluctuations similar to those in this paper, and one with a second long-run equilibrium position that proves to be attractive. The greatest task, however, will be an econometric estimation of the unobserved animal spirits, either in a model building block or an entire macro model. This problem could, for example, be addressed by using the Kalman filter or the method of simulated moments, respectively.

6 References


