

Transaction Taxes, Traders' Behavior and Exchange Rate Risks

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TRANSACTION TAXES, TRADERS' BEHAVIOR AND EXCHANGE RATE RISKS

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Abstract: We propose a new model of chartist-fundamentalist-interaction in which both groups of traders are allowed to select endogenously between different forecasting models and different investment horizons. Stochastic interest rates in both countries and different behavioral assumptions for trend-extrapolating and fundamental based forecasts determine the agents' market orders which drive the exchange rate. A numerical analysis of the model shows that it is able to replicate stylized facts of observed financial return time series like excess kurtosis and volatility clustering. Within this framework we study the effects of transaction taxes on exchange rate volatility and traders' behavior measured by their population fractions. Simulations yield the result that on the macroscopic level these taxes reduce the variance of exchange rate returns, but also increase their kurtosis. Moreover, on the microscopic level the tax harms short-term speculation in favor of long-term investment, while it also harms trading rules based on economic fundamentals in favor to trend extrapolating trading rules.

Key words: *Chartist-Fundamentalist-Interaction, Exchange Rates, Financial Market Volatility, Transaction Taxes*

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1 Introduction

Foreign exchange markets are excessively volatile and risky due to speculative bubbles and crashes. These transitory bubbles and crashes do not reveal rational arbitrage-free pricing behavior but might be due to irrational and trend-chasing behavior of speculators. Because trend-chasing behavior and short-term speculation leads to excessive risks, policy instruments like transaction taxes are proposed for reducing speculative attacks and excessive risks.

Survey data of foreign exchange markets yields empirical evidence of heterogeneous expectations among traders. Due to survey studies like the one conducted by TAYLOR, M. AND H. ALLEN (1992), these short term expectations are excessively volatile and display extrapolation behavior, while long term expectations are regressive and therefore of a stabilizing nature. Based on this empirical fact several studies like BROCK AND HOMMES (1997, 1998), CHIARELLA AND HE (2002), DEGRAUWE AND GRIMALDI (2006) and LUX AND MARCHESI (2000) start to incorporate heterogeneous expectations into economic models of exchange rate determination.

Because econometric tests on rational expectations in the foreign exchange market¹ are rejected and the efficient market approach cannot explain the stylized facts of financial market time series, researchers switched to the chartist-fundamentalist approach based on the empirical evidence of heterogeneous expectations from survey studies. This model framework is an alternative expectations hypothesis and an appealing building block for models of the foreign exchange market. It assumes that traders are bounded rational in that they do not use all available information and economic

¹See TAYLOR AND ALLEN (1992) and MENKHOFF (1997) among others.

models to forecast the exchange rate. Instead they rely on simple rules of thumb because they do not know the whole structure of the model. Most of these interacting agent models assume that the market is populated by two types of traders. The chartist trader type searches for patterns in past exchange rates like trends and trend reversals for forecasting future rates, while fundamentalist traders search for over- and undervaluations and expect them to be corrected in the future. Moreover, this approach allows agents to choose endogenously one of this two views of the world. The success of this model framework to explain stylized facts of financial markets like the exchange rate disconnect, excess volatility, volatility clustering and excess kurtosis encourages to elaborate on them. Moreover, there is also empirical evidence for the chartist-fundamentalist approach².

Studies like WESTERHOFF (2003) use the chartist-fundamentalist approach for analyzing the effects of market regulations in foreign exchange markets. Westerhoff finds that small transaction taxes lower exchange rate volatility while a high Tobin tax rate will lead to an increase. He explains this finding with the composition of chartists and fundamentalists in the population. Small transaction taxes make destabilizing chartism unprofitable and increase the fraction of fundamentalist traders which stabilizes the exchange rate. If the tax rate exceeds a certain threshold also fundamentalism will be unprofitable and the fraction of chartist traders will rise, so that this destabilizes the exchange rate again and volatility will rise.

MANNARO ET AL. (2005) find in their simulation study within an artificial stock market framework that volatility will fall by 2% for a tax rate of 0.1%,

²ENGLE AND HAMILTON (1990) find that there is regime switching in exchange rates in that there are phases of trends and mean-reversion. VIGFUSSON (1997) finds empirical evidence by estimating parameters of the chartist-fundamentalist model in a Markov-switching framework.

while it will fall by 8% for a tax rate of 0.5% with respect to the reference situation without taxes. Moreover, the percentage fraction of fundamentalists will rise due to the imposition of the transaction tax. In a simulation with only random traders and chartists a small tax can also lead to a small increase in volatility.

In this paper we want to introduce an extended version of the chartist-fundamentalist model for the foreign exchange market. Our model is similar to the models of BROCK AND HOMMES (1997, 1998), CHIARELLA AND HE (2002) and DEGRAUWE AND GRIMALDI (2006) among others. In contrast to these models we allow agents to choose between different investment horizons, such that there are short-term chartists and fundamentalists and long-term chartists and fundamentalists. Moreover, we also deviated from the commonly used discrete choice model for the evolution of trading rules and introduce another evolutionary mechanism that also allows to choose between different investment horizons. Simulations of the baseline model show that the model does well in replicating stylized facts like the unit-root property of exchange rates, clustering of return volatility and excess kurtosis in the distribution of returns.

The second task of our paper is to introduce transaction taxes into the model in order to analyze, how these taxes influence traders behavior and financial market risks. Simulations yield the result that on the microscopic level transaction taxes prevent long term traders to switch to short term speculation. On the macroscopic level these taxes reduce the variance of exchange rate returns but also increase their kurtosis. Moreover, the tax harms short-term speculation in favor of long-term investment, while it also harms trading rules based on economic fundamentals in favor to trend extrapolating trading rules.

The remainder of this paper is organized as follows: The next section presents the model economy, while section three will present the numerical analysis of the model, while section four concludes.

2 The Model Economy

The model is similar to that proposed by DE GRAUWE AND GRIMALDI (2006). Building blocks of the model are

- (i) the agents' portfolio selection problem,
- (ii) the agents' forecasts via different forecasting models,
- (iii) agents' evaluation of these portfolio rules by comparing their past profitability, and
- (iv) in our model the exchange rate is set by a market maker in contrast to DEGRAUWE AND GRIMALDI (2006) , while traders are also allowed to choose between different investment horizons.

2.1 Fundamental Factors and Arbitrage

In this model the fundamental factors driving the exchange rate are the gross rates of return on the domestic and foreign bond with one-period maturity. We assume both interest rates $R = (1 + r)$ to follow stochastic mean-reverting processes of the form

$$\ln R_t = (1 - \alpha) \ln \bar{R} + \alpha \ln R_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2), \quad (1)$$

where \bar{R} is the long-run average interest rate, $\alpha \in [0, 1]$ is the rate of mean-reversion and ε_t is a random innovation to the interest rate.

Analogue, the rate of return on the foreign one-period-bond follows

$$\ln R_t^* = (1 - \alpha) \ln \bar{R}^* + \alpha \ln R_{t-1}^* + \varepsilon_t^*, \quad \varepsilon_t^* \sim \mathcal{N}(0, \sigma^2). \quad (2)$$

Assuming homogeneous interest rate expectations and that all agents know the data generating processes for the interest rates, they can price the long-term bonds according to the expectations hypothesis of the term structure. The expectations hypothesis states that no arbitrage should be possible between the rates of return of a long-term bond and the rates of return of a sequence of one-period bonds over the maturity of the long-term bond. This gives us the following valuation formula for long-term bonds

$$\ln R_{t,N} = \frac{1}{N} \sum_{n=0}^N \mathbf{E}_t \ln R_{t+n}. \quad (3)$$

Using the fact that the n -period-ahead forecast of the autoregressive process for the interest rate is

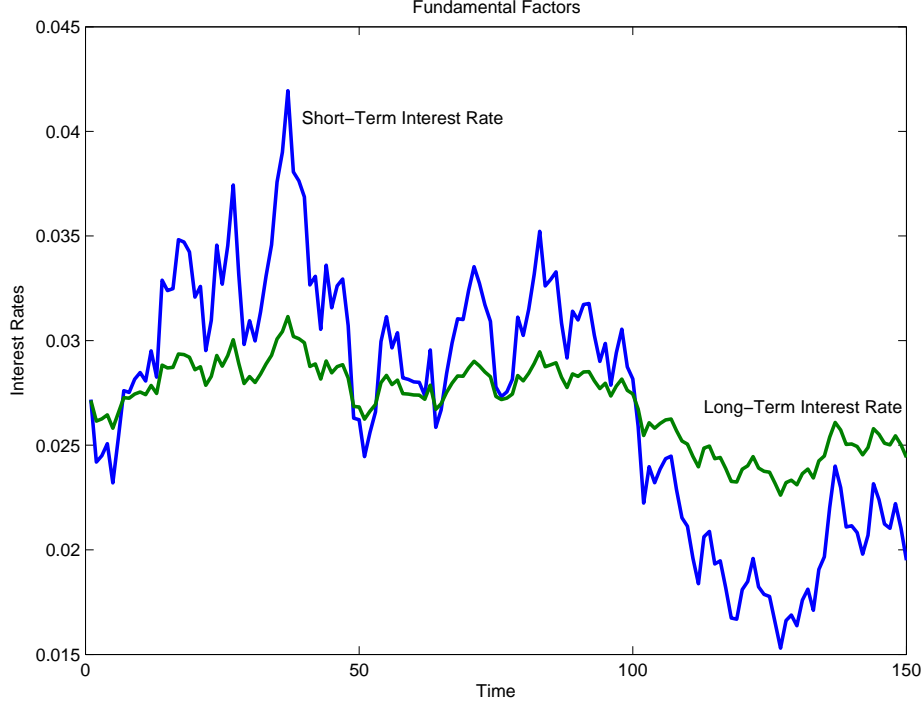
$$\mathbf{E}_t \ln R_{t+n} = \alpha^n \ln R_t + (1 - \alpha^n) \ln \bar{R} \quad (4)$$

and applying the rule for the finite geometric series yields the long-term interest rate

$$\ln R_{t,N} = \frac{1}{N} \left\{ \frac{1 - \alpha^N}{1 - \alpha} \cdot \ln R_t + n - \frac{1 - \alpha^N}{1 - \alpha} \ln \bar{R} \right\}. \quad (5)$$

FIGURE 1 shows the time series of short-term and long-term interest rates of a typical simulation run.

Fig. 1: Fundamental Factors



Note: Model generated time series from the baseline simulation. The used parameter values are those given in table 1. The first 1000 data points were removed.

Deviations from the no-arbitrage interest rate parity condition

$$\frac{E_t^i s_{t+n}}{s_t} = \left(\frac{R_{t,n}}{R_{t,n}^*} \right)^n \quad (6)$$

arise because interest rates follow stochastic processes. This deviation promises profits for foreign exchange traders and provokes them to demand foreign currency in the financial market. Note, that s_t is the bilateral exchange rate, while τ is the transaction tax rate. If this equation holds with equality the expected interest rate change will offset the interest rate differential and no trade will occur, because all profits are already arbitrated away.

2.2 Traders' Demand for Foreign Currency

Following DEGRAUWE AND GRIMALDI (2006) we assume that each agent can invest into a domestic asset and a foreign asset. In contrast to DEGRAUWE AND GRIMALDI (2006) both assets are risky due to the randomness of domestic and foreign interest rates and due to exchange rate risks.

We assume overlapping generations of traders, who enter the market for their pertinent investment horizon. Afterwards they will leave the market and consume their profits. The timing of each period is as follows:

- (i) trader i enters the market. He observes interest rates, the exchange rate and the past profits of the other traders. Depending on the past profits of the other traders, he decides to be a short-run or long-run fundamentalist or to be a short-run or long-run chartist trader,*
- (ii) depending on the interest rate differential and his expected depreciation of the exchange rate the trader decides how much to invest in the domestic and the foreign asset,*
- (iii) after the trader has realized his profit, he leaves the market and consumes.*

Agents are assumed to have preferences towards risks with constant absolute risk aversion characterized by the following utility function

$$U(W_{t+n}^i, \alpha_i) = -\exp\{-\alpha_i W_{t+n}^i\}, \quad (7)$$

where W_t^i is agent i 's wealth at time t , $n \in \{1, \dots, N\}$ is the agents' investment horizons, and α_i is the agents Arrow-Pratt measure of absolute risk

aversion. The agent's wealth is assumed to follow

$$W_{t+n}^i = (R_t^*)^n s_{t+n} d_t^i (1 - \tau)^2 + (R_t)^n (W_t^i - s_t d_t^i), \quad (8)$$

where $R = (1+r)$ and $R^* = (1+r^*)$ are the gross returns on the domestic and foreign bond, while s_t is the bilateral exchange rate between both countries. The tax rate for foreign exchange market transactions is denoted with $\tau \in [0, 1]$. The first part is the return on the foreign asset, while the second term measures the costs of borrowing in the domestic country. For $n = 1$ and $\tau = 0$ this budget constraint collapses to the one proposed by DeGrauwe and Grimaldi (2006).

If we assume wealth to be normally distributed we can simplify the portfolio selection problem by maximizing the certainty equivalent

$$U(W_t^i, \alpha_i) = \mathbf{E}_{t-1}^i[W_t^i] - \frac{\alpha_i}{2} \mathbf{Var}_{t-1}^i[W_t^i] \quad (9)$$

subject to the same budget constraint. Maximization yields the following demand function for agent i with investment horizon n

$$d_t^{i,n} = \frac{\mathbf{E}_t^i[W_{t+n}^i]}{\alpha_i \mathbf{Var}_t^i[W_{t+n}^i]} = \frac{(R_t^*)^n (1 - \tau)^2 \mathbf{E}_t^i[s_{t+n}] - (R_t)^n s_t}{\alpha \sigma_{i,t}^2}. \quad (10)$$

Thus, trader i 's demand is decreasing in his degree of risk aversion, in a higher risk $\sigma_{i,t}^2$, decreasing in the transaction tax rate τ , and increasing in the expected profit. For $n = 1$ and $\tau = 0$ the demand function collapses to the one used in DeGrauwe and Grimaldi (2006).

If we assume, following Brock and Hommes (1997) that the risk evaluation is the same for all agents and constant over time, the demand function

simplifies to

$$d_t^{i,n} = \psi \left((R_t^*)^n (1 - \tau)^2 \mathbb{E}_t^i [s_{t+n}] - (R_t)^n s_t \right). \quad (11)$$

2.3 Traders' Forecasting Models

We assume that the true data generating process for the exchange rate is unknown to the agents. Therefore they use ad-hoc rules for forecasting. We assume that two types of forecasting rules are used. A rule which reacts on trends in the exchange rate is commonly called chartist rule or technical trading rule. The other technique called fundamentalist forecasting rule looks for over- and undervaluations of the exchange rate with respect to its arbitrage free fundamental value and expects a reversion back to it.

The fundamentalist forecasting rule for the one-step-ahead prediction of the exchange rate can be written as

$$\mathbb{E}_t^f [s_{t+1} - s_t] = \kappa^f \cdot (s_t^f - s_t). \quad (12)$$

Thus, this rule predicts an exchange rate change such that $\kappa^f \cdot 100\%$ of the disequilibrium $s_t^f - s_t$, that is the deviation of the realized exchange rate s_t from the arbitrage-free exchange rate s_t^f , will be corrected by the subsequent exchange rate change. Note that the two step ahead forecast assumes that $\kappa^f \cdot 100\%$ of the remaining disequilibrium $(1 - \kappa^f) \cdot (s_t^f - s_t)$ will be corrected by the subsequent exchange rate change and so on. Thus, the n -step ahead forecast will be

$$\mathbb{E}_t^f [s_{t+n} - s_{t+n-1}] = \kappa^f (1 - \kappa^f)^{n-1} \cdot (s_t^f - s_t). \quad (13)$$

For $n = 1$ this forecasting model collapses to the one used in DEGRAUWE AND GRIMALDI (2006), LUX AND MARCHESI (2000), CHIARELLA AND HE (2002) and BROCK AND HOMMES (1997).

The expected exchange rate change $E_t^f[s_{t+n}]$ can be derived from the forecasted exchange rate changes as

$$\begin{aligned} E_t^f[s_{t+n}] - s_t &= E_t^f[s_{t+n} - s_{t+n-1}] + E_t^f[s_{t+n-1} - s_{t+n-2}] + \dots + E_t^f[s_{t+1} - s_t] \\ &= [1 - (1 - \kappa^f)^n] \cdot (s_t^f - s_t), \end{aligned} \quad (14)$$

where the explicit derivation can be found in the appendix.

Fundamentalists believe that the arbitrage-free exchange rate s_t^f is the exchange rate under which the uncovered interest rate parity condition holds with equality

$$s_t^f = s_{t-1} \cdot \frac{R_{t-1}}{R_{t-1}^*}. \quad (15)$$

Therefore, if s_t^f realizes, the exchange rate change offsets the possible profits from the interest rate differential and no arbitrage should be possible.

The technical forecasting rule for the one-step-ahead prediction can be specified as follows

$$E_t^c[s_{t+1} - s_t] = (\kappa^c) \cdot (s_t - s_{t-1}). \quad (16)$$

Thus, this forecasting model predicts a trend continuation. If the exchange rate change $s_t - s_{t-1}$ is one, than this forecasting model predicts the next exchange rate change to be κ^c . As usual in the theory of autoregressive models we use the last period's forecast to predict the next future exchange rate if we do not have information about realizations. Thus, the two-step-ahead forecast expects an exchange rate change of $(\kappa^c)^2$ and so on. Thus,

the n -step-ahead prediction will be

$$\mathbf{E}_t^c[s_{t+n} - s_{t+n-1}] = (\kappa^c)^n \cdot (s_t - s_{t-1}). \quad (17)$$

For $n = 1$ this forecasting model collapses to the one used in DEGRAUWE AND GRIMALDI (2006), LUX AND MARCHESI (2000), CHIARELLA AND HE (2002) and BROCK AND HOMMES (1997).

Equivalent to the fundamentalists' technique, chartists calculate the expected exchange rate change $\mathbf{E}_t^c[s_{t+n} - s_t]$ as

$$\mathbf{E}_t^c[s_{t+n} - s_t] = \mathbf{E}_t^c[s_{t+n} - s_{t+n-1}] + \dots + \mathbf{E}_t^c[s_{t+1} - s_t] \quad (18)$$

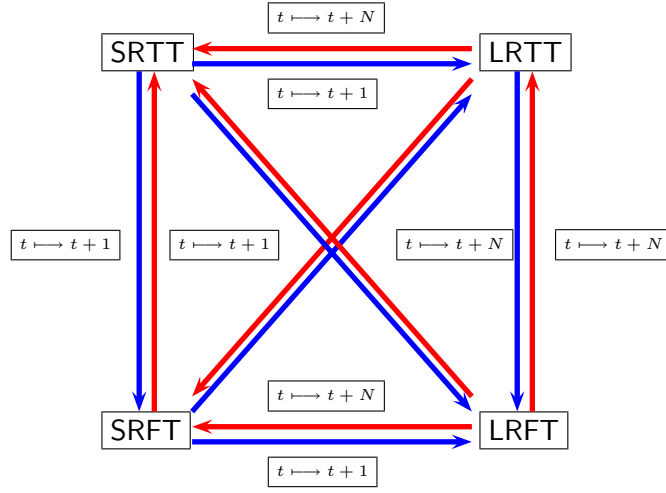
$$= \frac{1 - (\kappa^c)^n}{1 - \kappa^c} \cdot \kappa^c \cdot (s_t - s_{t-1}), \quad (19)$$

where the explicit derivation can be found in the appendix.

2.4 Evolution of Trading Rules

The agents' strategy space consists of five trading rules. The agent can either be a short-run fundamentalist or a short-run chartist, or the trader can be a long-term fundamentalist or a long-term chartist. The fifth possibility for the agents is to stay inactive, that means not to trade.

Fig. 2: Possibilities to Change Trading Strategies



Note: The abbreviation SRTT denotes short-run technical trader, while LRTT denotes long-run technical trader, while SRFT denotes short-run fundamental trader and LRFT long-run fundamental trader. The fifth alternative for traders is to stay inactive for one period which is not included in the graphic.

Because we assume that agents may have multi-period investment horizons, the information concerning the individual agents investment horizon is saved in the matrix Φ_t , which has the dimension $5 \times M$ and may have for example the following form

$$\Phi_t = \begin{bmatrix} 1 & 1 & 100 & 84 & 1 \\ 1 & 1 & 61 & 31 & 1 \\ 1 & 1 & 87 & 54 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \quad (20)$$

The first two columns of this matrix identify the short run fundamentalist and the short run chartist, who have an investment horizon of one by construction. Columns three and four identify the long term fundamentalist and the long term chartist and the time until their investment matures. Agents are allowed to stay inactive for one period. This information is given in column five. Agents $1, \dots, M$ are given in rows. Thus, this matrix reads as

follows. If agent 1 is a long run fundamentalists, then the time to maturity of his investment is 100 periods. If he is a long term chartist, then the time to maturity is 84 periods. This matrix is updated as follows

$$\Phi_{t+1} = \Phi_t - [\mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{1}, \mathbf{0}], \quad (21)$$

where $\mathbf{0}$ is a $1 \times M$ vector of zeros, $\mathbf{1}$ is a $1 \times M$ vector of ones and M is the number of agents. Thus, the investment horizon of long term agents decreases by one period until maturity is reached. After that it switches back to the maximum investment horizon of N periods. The starting value for this updating process is generated by a random draw for the columns three and four.

Agents are only allowed to change their trading rules when maturity is reached. Thus short-term traders and inactive traders are allowed to switch every period, long term traders are not allowed to switch for N periods. The information about which agent is allowed to switch is contained in the matrix \mathbf{S}_t , with

$$\mathbf{S}_t(i, j) = 1 \iff \Phi_t(i, j) = 1 \quad (22)$$

$$\mathbf{S}_t(i, j) = 0 \iff \Phi_t(i, j) \neq 1. \quad (23)$$

Thus, if $\mathbf{S}_t(i, j) = 1$ then agent i is allowed to change his trading rule, if he is type j . If $\mathbf{S}_t(i, j) = 0$, then agent i with trading rule j is not allowed to change his type. Because this matrix only contains information if an agent is allowed to switch or not. The matrix tells us for example that agent one is allowed to change his type, if he is type one, two or three, but he is not allowed to switch if he is type four. Thus, this matrix does not tell us, which trading rule the agent is currently using. This information is contained in

the matrix $\mathbf{\Gamma}_t$, where $\mathbf{\Gamma}_t(i, j)$ is one if agent i uses the trading rule j and zero otherwise. Thus, the row sum of this matrix is one, because an agent can only use one trading rule at the same time.

We assume that agents switch to the trading rule, which was the most successful in the past if they are allowed to switch. Therefore agents calculate the profits each trading rule yielded over the last N periods. The vector of past profits is given by

$$\mathbf{\Pi}_t = [\pi_t^{f,1}, \pi_t^{c,1}, \pi_t^{f,N}, \pi_t^{c,N}, 0], \quad (24)$$

where the agents realizes a profit of zero if he stays inactive.

The profit of agent i is measured by the variable $\pi_t^{i,n}$

$$\pi_t^{i,n} = \{s_t(R_{t-n}^*)^n(1 - \tau)^2 - s_{t-n}(R_{t-n})^n\} \cdot d_t^{i,n}, \quad (25)$$

for $i \in \{c, f\}$ and $n = 1, \dots, N$.

Table 1: Cash Flows of Short-term and Long-term Traders

Short-term Trader				
time	t	$t + 1$...	$t + N$
	$d_t^{i,1}$	$-d_t^{(i,1)} \left((1 - \tau)^2 (1 + R_t^*) S_{t+1} \right)$	—	—

Long-term Trader				
time	t	$t + 1$...	$t + N$
	$d_t^{i,N}$	—	—	$-d_t^{(i,N)} \left((1 - \tau)^2 (1 + R_{t,N}^*)^N S_{t+N} \right)$

Here we replaced the forecast $E_{t-n}^i s_t$ with the realized exchange rate s_t .

Thus, $\pi_t^{i,n}$ measures the profit per unit currency that results from the ex-

change rate change and the interest rate differential times the amount of currency demanded. This expression is similar to the one used in Grimaldi (2004) and DeGrauwe and Grimaldi (****) with the difference that we scale the profit per unit currency by the currency demanded by the agent.

The vector $\mathbf{\Pi}^*_t$ has the entry one at the same place, where $\mathbf{\Pi}_t$ has its maximum and zeros at all other entries. Thus, this vector indicates to which trader type the agent has to switch if he is allowed to switch. The switching of agents is conducted, by replacing the pertinent row in the matrix $\mathbf{\Gamma}_t$ with the vector $\mathbf{\Pi}_t$. This operation is conducted if an agent is allowed to switch. This is possible if the condition

$$\mathbf{S}(i, j) = 1 \quad \& \quad \mathbf{\Gamma}(i, j) = 1 \quad (26)$$

holds. If

$$\mathbf{S}(i, j) = 0 \quad \& \quad \mathbf{\Gamma}(i, j) = 1, \quad (27)$$

then $\mathbf{\Gamma}(i, j) = 1$, that means, the agent is not allowed to switch and has to use his old trading strategy. In all other cases the matrix $\mathbf{\Gamma}(i, j)$ has the entry zero.

The information about the number of agents, who are allowed to trade and the number of agents being using one special trading rule is contained in these matrices.

2.5 Institutional Properties and Price Setting

The market maker collects all individual demands in order to determine the market demand. Individual demands $d_t^{i,n}$ can be aggregated to the market demand D_t by adding them, while weighting them with the population

fractions $w_t^{i,n}$ of traders, who are allowed to trade

$$D_t = w_t^{c,1} d_t^{c,1} + w_t^{c,N} d_t^{c,N} + w_t^{f,1} d_t^{f,1} + w_t^{f,N} d_t^{f,N} \quad (28)$$

$$- w_{t-1}^{c,1} \pi_{t-1}^{c,1} - w_{t-N}^{c,N} \pi_{t-N}^{c,N} - w_{t-1}^{f,1} \pi_{t-1}^{f,1} - w_{t-N}^{f,N} \pi_{t-N}^{f,N}. \quad (29)$$

Agents are allowed to trade at the beginning of their investment and at the end of their investment. They have to pay back the loan they raised in order to invest which is denoted in their home currency and because they want to consume in their home country. The last effect is captured by the last term in this equation.

If market demand is positive, the market maker will rise the price of the exchange rate, while he will lower it, if market demand is negative. Thus the exchange rate changes proportional to the sum of all market orders.

The behavior of the market maker can be approximated by the following price impact function³

$$s_{t+1} = s_t + \beta s_t D_t. \quad (30)$$

The exchange rate return can be calculated as

$$\rho_{t+1} = \frac{s_{t+1} - s_t}{s_t} = \beta D_t. \quad (31)$$

Thus, the model is complete now. Because it cannot be solved analytically, we will rely on results derived by numerical simulations in the next section.

³Kyle (1985) derives this price impact function as the solution of his continuous double auction model. Lux and Marchesi (2000) and Westerhoff (2003) also use this pricing rule within an agent-based-framework.

3 Non-Stochastic Steady States

At the steady state all shocks will be zero and all variables will be constant.

Thus a steady state is characterized by

$$\varepsilon_t = \varepsilon_t^* = 0 \quad (32)$$

and

$$R_t = R_{t-1} = \bar{R}, R_t^* = R_{t-1}^* = \bar{R}^*, s_t = s_{t-1} = \bar{s}, \quad (33)$$

$$d_{i,t} = d_{i,t-1} = 0, \pi_{i,t} = \pi_{i,t-1} = 0, \quad (34)$$

while the population fractions are undetermined.

Summing up, the steady state is characterized by equal rates of return in both countries and no exchange rate change. Therefore we get zero demands and zero profits, because the exchange rate equals its no-arbitrage fundamental value.

4 Simulation Results

The model is simulated with the parameters given in TABLE 2. For the baseline simulation we set the transaction tax rate to zero in order to have a benchmark for the policy simulations conducted later.

Table 2: Calibrated Parameters for Baseline Simulation

Model Parameters			
<i>mean reversion parameters</i>		<i>standard deviations of shocks</i>	
$\alpha_1 = 0.96$	$\alpha_2 = 0.96$	$\sigma_1 = 0.03$	$\sigma_2 = 0.03$
<i>risk aversion parameters</i>		<i>max. horizon simulation length</i>	
$\delta^C = 1$	$\delta^F = 1$	$N = 100$	$T = 100\,000$
<i>exchange rate response</i>		<i>forecasting parameters</i>	
$\beta = 0.01$		$\kappa^C = 0.9$	$\kappa^F = 0.8$
<i>transaction tax</i>		<i>number of agents</i>	
$\tau = 0$		300	

Note: These parameters are used for the baseline simulation of the model without transaction taxes.

We assume the interest rates in both countries to be quite persistent because empirical exchange rate data are also quite near a unit-root process. Thus, we assume the two interest rate processes to follow

$$\ln R_t = 0.04 \cdot 1.005 + 0.96 \cdot \ln R_{t-1} + 0.03 \cdot \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1). \quad (35)$$

For the risk aversion we assume chartist traders and fundamentalist traders to have the same value for the Arrow-Pratt measure of absolute risk aversion. Moreover, we assume chartists to have an extrapolation parameter less than one

$$\mathbb{E}_t^C s_{t+n} - s_t = 0.9^n (s_t - s_{t-1}), \quad (36)$$

such that their forecasting model predicts a return of 0.9 for the next period, if the current return is one and the two-period return to be 0.81.

For the fundamentalist traders we assume that they expect exchange rate disequilibria to be corrected with 80% per period. Thus, their forecasting model becomes

$$\mathbb{E}_t^F s_{t+n} - s_t = 0.8 \cdot 0.2^{n-1} \cdot (s_t^f - s_t). \quad (37)$$

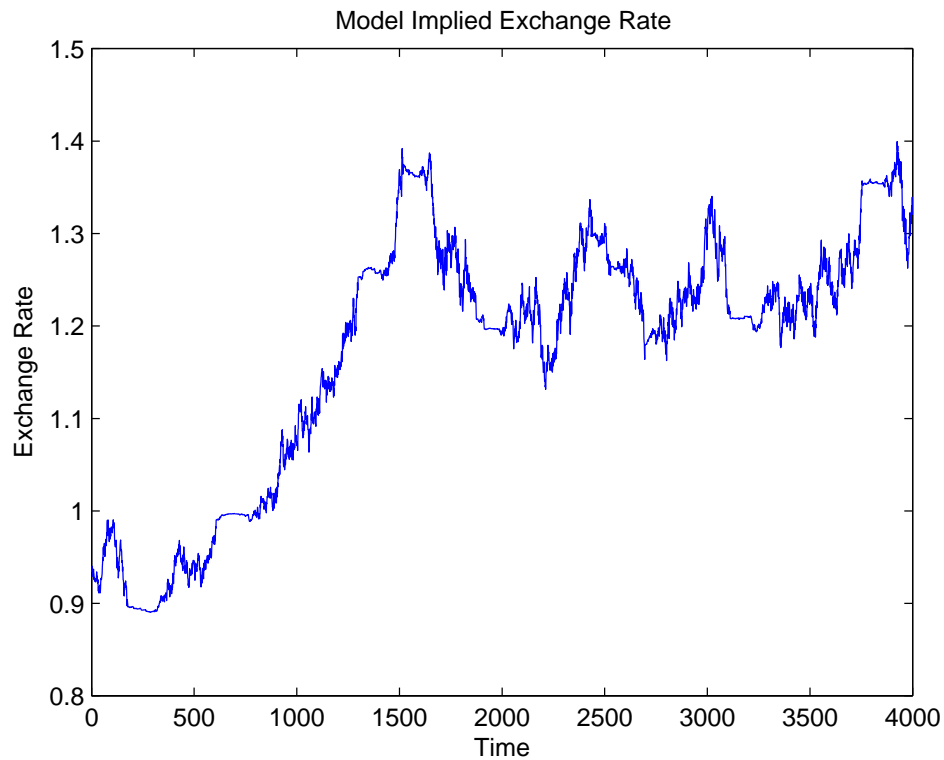
Furthermore, we set the exchange rate response to

$$s_{t+1} = s_t + 0.01 \cdot s_t D_t. \quad (38)$$

4.1 The Baseline Simulation

FIGURES 3 AND 4 show the simulation outcome of the baseline model without taxes. The exchange rate shows a random walk like behavior like empirical financial time series. One can clearly see that the time series displays periods of trends and crashes as we typically find in financial market time series.

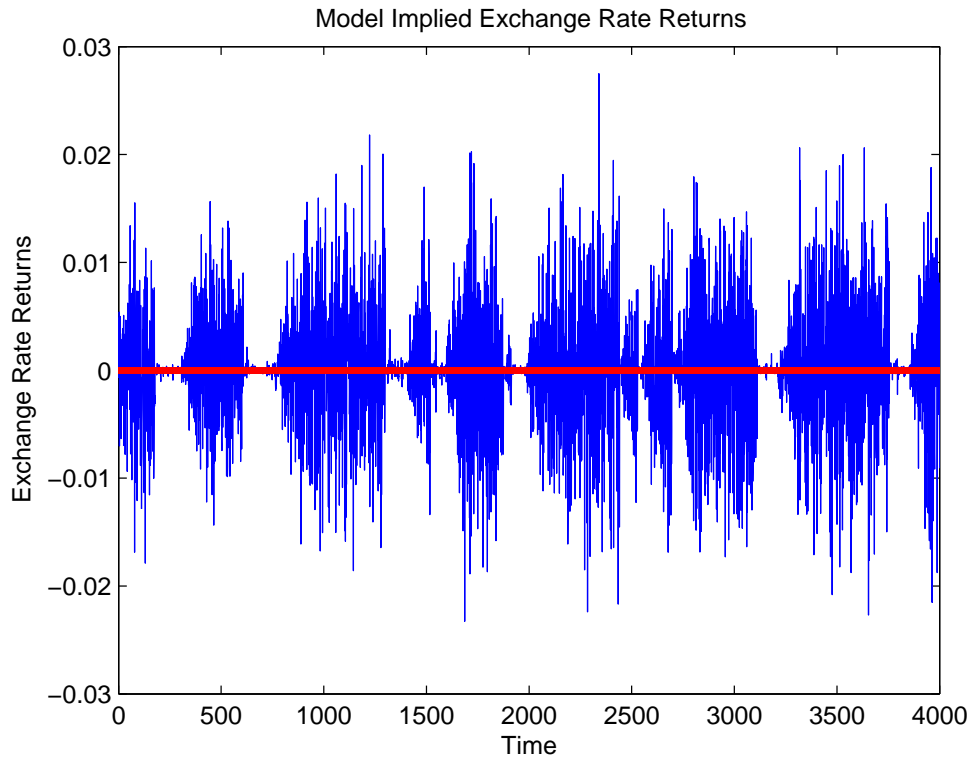
Fig. 3: Exchange Rate: Baseline Simulation



Note: Model generated time series from the baseline simulation. The used parameter values are those given in TABLE 2. The first 1000 data points were removed.

A second stylized fact which the model is able to reproduce is volatility clustering and excess kurtosis which can be seen from FIGURE 4.

Fig. 4: Exchange Rate: Baseline Simulation

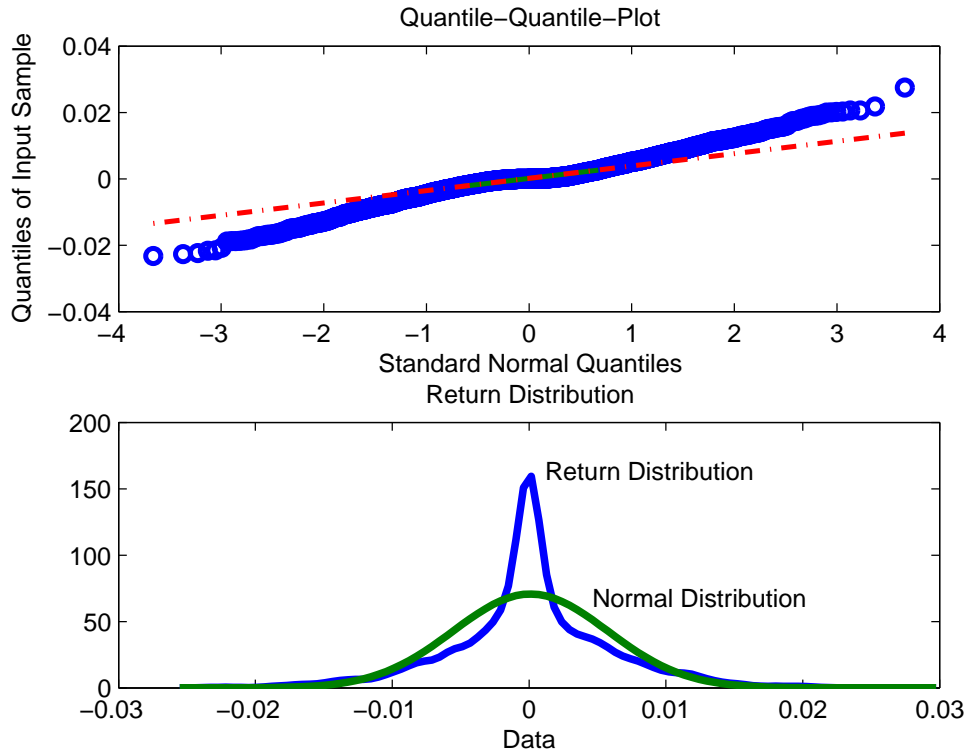


Note: Model generated time series from the baseline simulation. The used parameter values are those given in TABLE 2. The first 1000 data points were removed.

Both, in empirical time series as well as in the model produced time series periods of high volatility and periods of low volatility tend to cluster together. Moreover as can be seen in the figure is, that extreme returns are realized quite frequently.

By looking at FIGURE 5 we can analyze this phenomenon in greater detail. The upper subfigure shows a quantile-quantile-plot with respect to the normal distribution. Here quantiles of the standard normal distribution are plotted against the quantiles of the empirical return distribution. If the data is normally distributed all points should lie on the 45° line.

Fig. 5: Return Distribution: Baseline Simulation

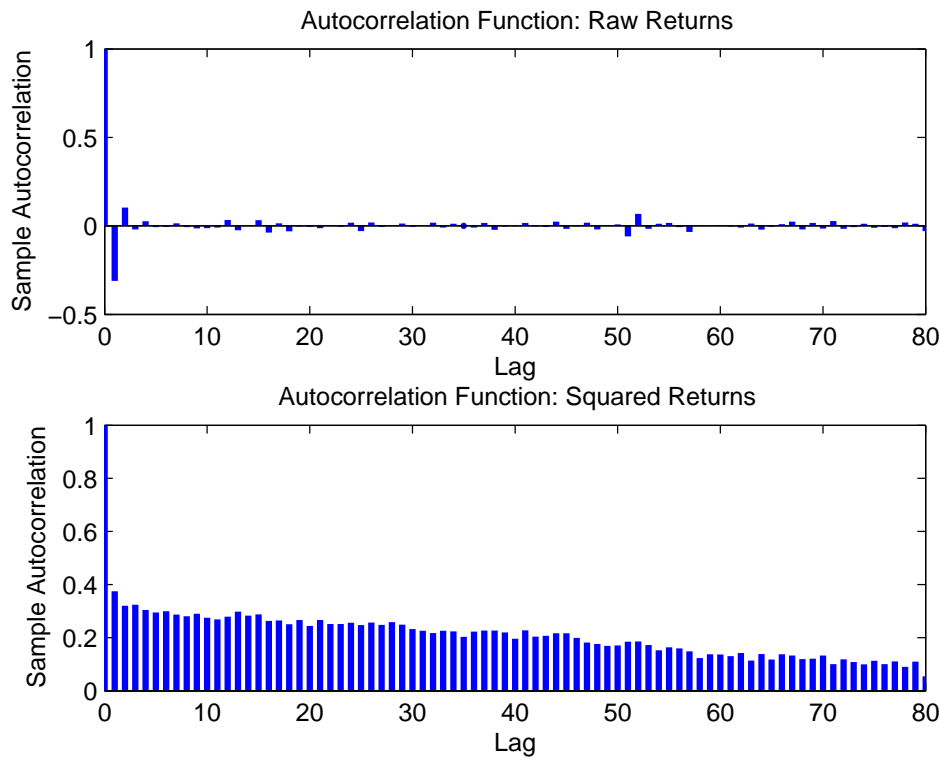


Note: Model generated time series from the baseline simulation. The used parameter values are those given in table 1. The blue line represents the kernel density of the model generated exchange rate returns, while the green line is the density of a normally distributed random variable with the same mean and the same variance. The first 1000 data points were removed. The used parameter values are those given in TABLE 2.

From this figure we can see deviations from the normal distribution in the positive and negative extreme parts. In the lower subfigure the estimated kernel density of the returns is plotted together with the density of a normally distributed random variable with the same mean and variance as the input sample for comparison. From this figure can be seen that the density of the model generated data has a higher peak and fatter tails with respect to the normal distribution which means that this distribution is leptokurtic.

The phenomenon of volatility clustering can be analyzed in more detail from FIGURE 5.

Fig. 6: Autocorrelation Functions: Baseline Simulation



Note: Model generated time series from the baseline simulation. The used parameter values are those given in TABLE 2. The first 1000 data points were removed.

FIGURE 6 plots the autocorrelation function of returns and squared returns for 100 lags. Here, raw returns display only small serial correlation which means that exchange rate returns are not predictable from their past data. This finding is in line with the efficient market hypothesis. In contrast to this squared returns display strong correlations over 100 lags. This indicates that although returns themselves are uncorrelated they are not independently distributed because squared returns display high serial dependencies. We

can interpret squared returns as a noise measure for volatility because

$$\text{Var}[r_t|\mathcal{I}_{t-1}] = \text{E}[r_t^2|\mathcal{I}_{t-1}] \text{ and } r_t^2 = \text{E}[r_t^2|\mathcal{I}_{t-1}] + v_t \quad (39)$$

$$\implies r_t^2 = \text{Var}[r_t^2|\mathcal{I}_{t-1}] + v_t. \quad (40)$$

Therefore, high serial correlations of squared returns indicates that volatility is serially correlated and therefore predictable. Small correlations in returns and large correlations in squared returns can also be found in empirical data as you can see in FIGURE 6. Thus, our model is also able to replicate this stylized fact of financial data.

Table 3: Summary Statistics of the Baseline Simulation

	Model	USD-Euro	YEN-USD	GBP-USD
<i>mean</i>	0.000	0.000	0.000	0.000
<i>st. deviation</i>	0.006	0.006	0.007	0.006
<i>skewness</i>	-0.021	0.014	-0.487	-0.135
<i>kurtosis</i>	4.549	3.619	7.335	6.573
<i>ARCH</i>	0.285	0.014	0.056	0.065
<i>GARCH</i>	0.715	0.977	0.942	0.922

Note: Mean, variance, skewness and kurtosis are calculated from the model generated exchange rate return data by using the parameters given in TABLE 2. ARCH and GARCH are the coefficients of an GARCH(1,1) model fitted to the model generated return data. The exchange rate data used in columns 3,4 and 5 are taken from the FRED2 database of the Federal Reserve Bank of St. Louis in daily frequency. The data is available under the series-ID: DEXUSEU, DEXJPUS, and DEXUSUK.

TABLE 3 contains summary statistics of the baseline simulation in comparison with summary statistics of empirical exchange rate return data. The mean of simulated returns and empirical returns is always zero, while the variance of the model equals the empirical returns because of the model calibration. The kurtosis of empirical data and of the baseline simulation is always greater than 3, which is the kurtosis of a normally distributed random variable. This fact also could be seen from the quantile-quantile-plot and the kernel density graphs. Moreover, we fitted a GARCH(1,1) model to the baseline simulation data and the empirical data. The GARCH-model due to BOLLERSLEV (1996) assumes the data to be conditional normally distributed

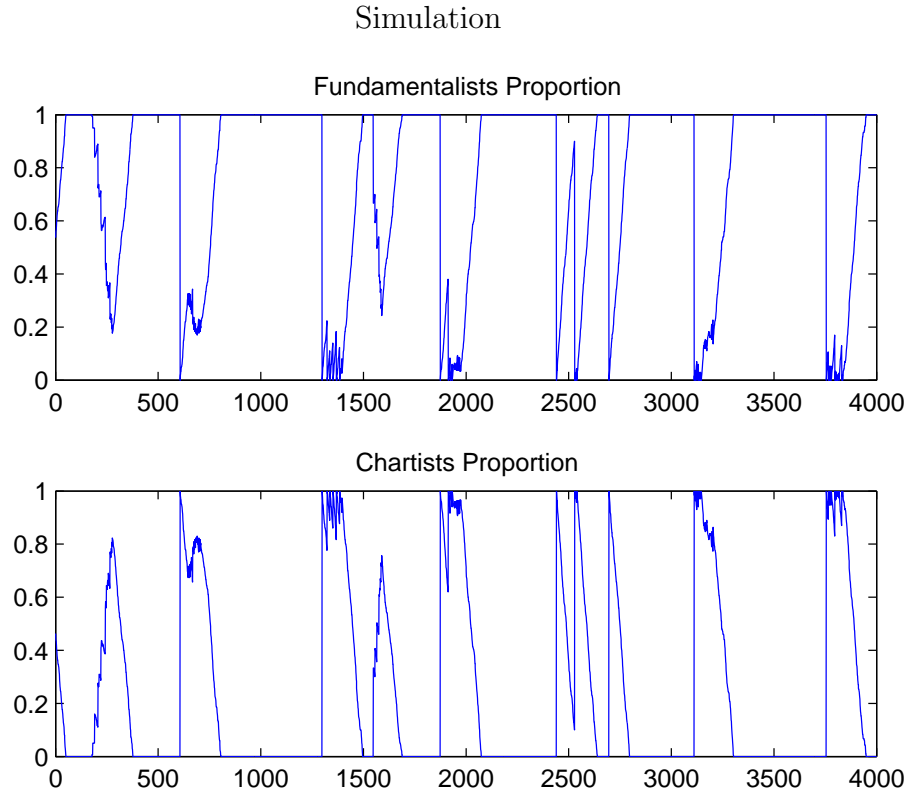
$$r_t | \mathcal{I}_{t-1} \sim \mathcal{N}(0, \sigma_t^2), \quad (41)$$

while the variance is assumed to follow an autoregressive process

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2. \quad (42)$$

New information about volatility can enter the model through squared returns, while the last term measures the persistency of volatility. Empirical studies usually find α to be less than 0.1 and β approximately 0.9, with $\alpha + \beta$ close to one. This is an indication of the strong persistency in volatility. From TABLE 3 you can infer, that this fact can also be found in estimates for the three exchange rate return time series as well as for the model generated return time series. Thus, our model is also able to replicate this stylized fact.

Fig. 7: Fraction of Fundamentalist and Technical Traders: Baseline

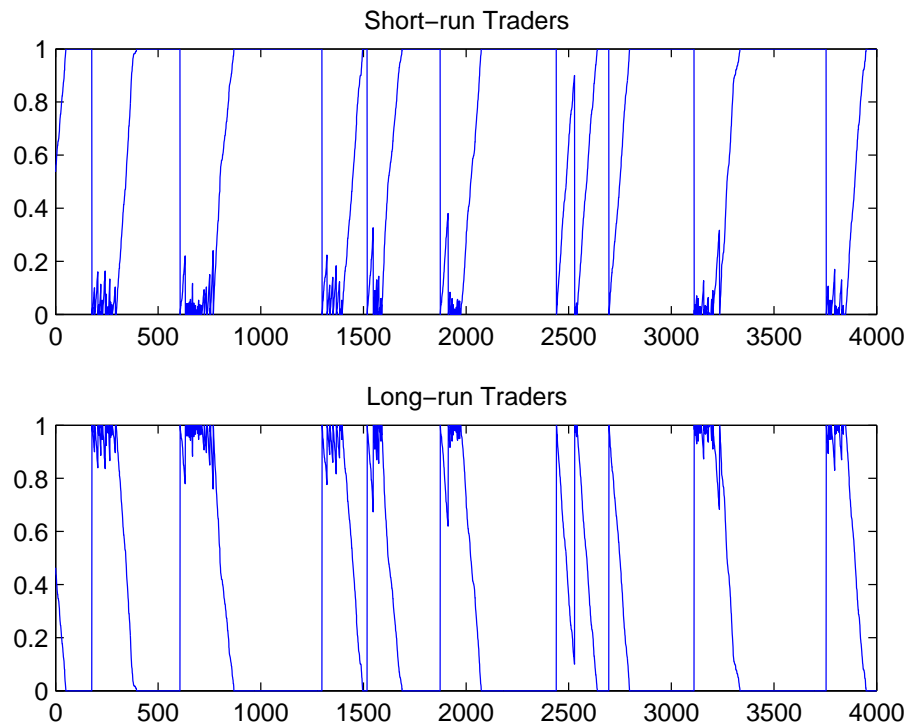


Note: Model generated time series from the baseline simulation. The used parameter values are those given in TABLE 2. The first 1000 data points were removed.

FIGURE 7 plots the evolution of the population fractions of traders using the technical trading rules and the fundamental trading rules. We can clearly see from this figure that from time to time majorities for one of this two trading rules emerge. It seems that the system is switching between states in which one of the two rules is used by all traders and that their view changes surprisingly. This is an indication of herding behavior among traders. If we compare this figure with FIGURE 4 we see that the periods in which one of the two trading rules dominates the market correspond to the high volatility and low volatility period in the exchange rate returns.

Thus, herding behavior is a source of the volatility clusters produced by the model.

Fig. 8: Fraction of Short-run and Long-run Traders: Baseline Simulation



Note: Model generated time series from the baseline simulation. The used parameter values are those given in TABLE 2. The first 1000 data points were removed.

FIGURE 8 plots the time variation of population fractions of traders having either short-term investment horizons or long-term investment horizons. Similar to FIGURE 7 the system is switching between the two views of the world. Thus, the market is either dominated by long-term traders or by short-term traders. The dominance of short-term traders is an indication of speculative attacks on one currency. If we compare this figure to FIGURE 4 we can again see that speculative attacks correspond to high volatility periods in the exchange rate returns.

All in all, this section showed, that our model is able to reproduce deviations of the exchange rate from the fundamental value, random walk or martingale behavior of the exchange rate, volatility clustering and fat tails in the distribution. Moreover, we learn about traders behavior that our financial market is characterized by herding behavior of traders. The market is either dominated by traders using trend-extrapolating trading rules or trading rules based on economic fundamentals. Moreover we see that the market is characterized by periods dominated by long-term or by short-term traders. This is an indication of speculative attacks on one currency.

The success of our model in replicating stylized facts of financial data encourages to use it for economic policy analysis by varying the transaction tax rate in order to analyze the effects of transaction taxes on financial risks. This we will do in the next section.

4.2 Sensitivity to Transaction Tax Rate Changes

4.2.1 Statistical Properties of the Exchange Rate

TABLE 4 shows summary statistics of the model generated exchange rate returns for different values of the transaction tax rate.

Table 4: Variation of the Transaction Tax

τ	0%	1%	2%	4%	6%
<i>mean</i>	0.000	0.000	0.000	0.000	0.000
<i>SE</i>	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
<i>variance</i>	0.005	0.004	0.003	0.002	0.002
<i>SE</i>	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)
<i>skewness</i>	0.051	0.084	0.206	0.369	0.425
<i>SE</i>	(0.102)	(0.242)	(0.481)	(0.774)	(0.817)
<i>kurtosis</i>	3.617	5.923	9.911	15.048	16.667
<i>SE</i>	(0.649)	(4.178)	(6.787)	(8.929)	(9.632)

Note: The remaining parameters are set to the values given in TABLE 2. The statistics are averages of 100 simulation runs of size 1000. The used parameter values are those given in TABLE 2. Standard errors are reported in parenthesis.

The statistics reported in this table are averages over 100 simulation runs of size 1000. From the table one can infer that the mean exchange rate return does not change due to changes in the transaction tax rate, while their variance is monotonically declining. Moreover one can see that although positive transaction tax rates reduce the variance of exchange rate returns they rise their kurtosis. Thus, positive transaction tax rates increase the probability of extreme positive and negative returns. This limits the success of taxes to reduce risks in foreign exchange markets.

4.2.2 Fundamental Traders and Technical Traders

From TABLE 5 one can infer how positive transaction tax rates influence traders behavior. The numbers belonging to this table are also averages over 100 simulation runs of size 1000 and are based on the same seed of random numbers like the statistics in TABLE 3. From this table one can infer that the number of traders using the fundamental trading rules is decreasing in the transaction tax rate while the number of traders using chartist rules is increasing. Moreover, the number of traders staying inactive are rising slightly in the transaction tax rate. Thus, under positive transaction tax rates chartist rules are more profitable than fundamental trading rules which is a contradiction to the conventional view of the proponents of a securities transaction tax who propose that traders will rely more on economic fundamentals under positive tax rates.

Table 5: Average Percentage Fractions of Used Trading Rules

τ	0%	1%	2%	4%	6%
<i>fundamental</i>	0.875	0.847	0.787	0.750	0.756
<i>SE</i>	(0.107)	(0.102)	(0.085)	(0.080)	(0.070)
<i>technical</i>	0.125	0.152	0.212	0.249	0.242
<i>SE</i>	(0.107)	(0.101)	(0.085)	(0.080)	(0.070)
<i>inactive</i>	0.000	0.001	0.001	0.001	0.002
<i>SE</i>	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
<i>short-term</i>	0.868	0.633	0.413	0.256	0.228
<i>SE</i>	(0.113)	(0.217)	(0.197)	(0.127)	(0.110)
<i>long-term</i>	0.132	0.366	0.586	0.743	0.770
<i>SE</i>	(0.113)	(0.217)	(0.197)	(0.127)	(0.110)
<i>inactive</i>	0.000	0.001	0.001	0.001	0.002
<i>SE</i>	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)

Note: Average percentage fractions of used trading rules during a typical simulation run for different transaction tax rates τ . The results of each column are based on the same seed of random variables. The statistics are averages of 100 simulation runs of size 1000. The used parameter values are those given in TABLE 2. Standard errors are reported in parenthesis.

4.2.3 Short-term and Long-term Traders

From TABLE 5 one can also infer that the number of short term traders is decreasing in the transaction tax rate while the number of long term traders is increasing. This is in line with the conventional view that a transaction tax makes short term trading more costly and therefore prevents speculative attacks in favor of long term investments.

5 Conclusion

This study wants to analyze the effectiveness of a transaction tax within an agent-based framework. We propose a new model for the foreign exchange market with four types of agents: short- and long term fundamentalists and short- and long-term chartists. Stochastic interest rates in both countries lead to temporal arbitrage opportunities and therefore to demand for foreign currency. A market maker aggregates the agents' market orders and rises the exchange rate due to positive excess demand and lowers it due to negative excess demand.

Simulations of the baseline model without transaction taxes produce time series with realistic time series properties like in empirical exchange rate data. This means that the model is capable to reproduce stylized facts of financial variables like the unit root property, volatility clustering and excess kurtosis. A comparison with empirical data shows that the model is able to replicate these stylized facts very well. Moreover, our financial market is characterized by periods dominated by traders using trend-extrapolating trading rules or by trading rules based on economic fundamentals. Furthermore, periods emerge which are dominated either by short-term speculators or by long-term investors. This is an indication of sudden speculative attacks on one currency.

The economic policy analysis of our model shows that positive transaction taxes are capable of reducing volatility. The disadvantage of this policy instrument is, that the probability of extreme positive or negative exchange rate returns is increased. That means higher transaction tax rate increases the kurtosis of the return distribution. The tax alters traders behavior by reducing short-term speculation in favor of long-term investments, which is

in line with the arguments of the proponents of the Tobin tax. In contrast to their view, in our model the tax favors trend extrapolating trading rules and punishes trading rules based on economic fundamentals. Because trend extrapolating trading rules are a source of destabilization of the exchange rate, this can be the reason why the transaction tax increases the kurtosis of the return distribution.

Summing up, further research should look for analytical solutions to a simplified version of this model and for extensions by the incorporation of other long-term investment strategies into the model in order to get more information about the effectiveness of transaction taxes on traders' behavior and the reduction of risks in financial markets.

6 Appendix: Derivation of the Multi-Period Forecasts

6.1 Fundamentalists' Forecasts

The one-step-ahead forecast of the future change in the exchange rate is given by

$$\mathbb{E}_t^F [s_{t+1} - s_t] = \kappa^f (s_t^f - s_t). \quad (43)$$

Thus, the fundamentalists' forecast yields, that $(1 - \kappa^f)(s_t^f - s_t)$ disequilibrium will remain. Thus, the next predicted exchange rate change will be

$$\mathbb{E}_t^F [s_{t+2} - s_{t+1}] = \kappa^f (1 - \kappa^f) (s_t^f - s_t). \quad (44)$$

Again, $(1 - \kappa^f)^2(s_t^f - s_t)$ disequilibrium will remain. In general, we have

$$\mathbf{E}_t^F [s_{t+n} - s_{t+n-1}] = \kappa^f (1 - \kappa^f)^{n-1} (s_t^f - s_t). \quad (45)$$

If fundamentalists want to forecast the exchange rate s_{t+n} , they have to forecast the future exchange rate changes as we have done before and then to calculate

$$\begin{aligned} \mathbf{E}_t^f s_{t+n} &= s_t + \mathbf{E}_t^f [s_{t+1} - s_t] + \mathbf{E}_t^f [s_{t+2} - s_{t+1}] + \dots + \mathbf{E}_t^f [s_{t+n} - s_{t+n-1}] \quad (46) \\ &= s_t + \kappa^f (s_t^f - s_t) + \kappa^f (1 - \kappa^f) (s_t^f - s_t) + \dots + \kappa^f (1 - \kappa^f)^{n-1} (s_t^f - s_t) \\ &= s_t + \left(\kappa^f (1 - \kappa^f)^0 + \kappa^f (1 - \kappa^f) + \dots + \kappa^f (1 - \kappa^f)^{n-1} \right) (s_t^f - s_t) \\ &= s_t + \kappa^f \left((1 - \kappa^f)^0 + (1 - \kappa^f) + \dots + (1 - \kappa^f)^{n-1} \right) (s_t^f - s_t) \end{aligned}$$

By applying the rule for the geometric series, we can write this as

$$\mathbf{E}_t^f s_{t+n} = s_t + \left(1 - (1 - \kappa^f)^n \right) (s_t^f - s_t). \quad (47)$$

6.2 Chartists' Forecasts

The one-step-ahead forecast of the next exchange rate change is given by

$$\mathbf{E}_t [s_{t+1} - s_t] = \kappa^c (s_t - s_{t-1}), \quad (48)$$

while the one-step-ahead forecast for the subsequent exchange rate change is given by

$$\begin{aligned} \mathbf{E}_t[s_{t+2} - s_{t+1}] &= \kappa^c \mathbf{E}_t^c(s_{t+1} - s_t) \\ &= (\kappa^c)^2 (s_t - s_{t-1}). \end{aligned} \quad (49)$$

In general, we have

$$\mathbf{E}_t[s_{t+n} - s_{t+n-1}] = \kappa^c \mathbf{E}_t^c(s_{t+n-1} - s_{t+n-2}) \quad (50)$$

$$= (\kappa^c)^n (s_t - s_{t-1}). \quad (51)$$

For forecasting the future exchange rate s_{t+n} chartists have to forecast the exchange rate changes as we have done before and then to calculate

$$\begin{aligned} \mathbf{E}_t^c s_{t+n} &= s_t + \mathbf{E}_t^c[s_{t+1} - s_t] + \mathbf{E}_t^c[s_{t+2} - s_{t+1}] + \dots + \mathbf{E}_t^c[s_{t+n} - s_{t+n-1}] \\ &= s_t + \kappa^c (s_t - s_{t-1}) + (\kappa^c)^2 (s_t - s_{t-1}) + \dots + (\kappa^c)^n (s_t - s_{t-1}) \\ &= s_t + \left(\kappa^c + (\kappa^c)^2 + \dots + (\kappa^c)^n \right) (s_t - s_{t-1}) \\ &= s_t + \kappa^c \left((\kappa^c)^0 + (\kappa^c)^1 + \dots + (\kappa^c)^{n-1} \right) (s_t - s_{t-1}) \end{aligned} \quad (52)$$

By applying the formula for the geometric series we get

$$\mathbf{E}_t^c s_{t+n} = s_t + \kappa^c \cdot \frac{(1 - \kappa^c)^n}{1 - \kappa^c} \cdot (s_t - s_{t-1}). \quad (53)$$

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