Can Monetary Policy Tame Harrodian Instability?

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Abstract

The paper introduces monetary policy into the canonical Kaleckian growth model with Harrodian instability. It abstains, however, from the simple and immediately stabilizing interest rate inverse IS curve. Instead, more indirect effects are examined, which realistically will take time to work out. In particular, (i) the trend rate of growth governing the investment decisions additionally responds to the difference between the profit rate and the real rate of interest; and (ii) the real interest rate may enter dynamic adjustments of the price markup. The main finding is that the Harrodian forces could still be overcome and stability of the steady state position is re-established provided that the profitability motive in (i) and the responsiveness in the Taylor policy rule are both sufficiently strong. By contrast, the indirect feedback effects produced by (ii) broaden the scope for instability. In sum, monetary policy in this extended framework can favour stability but is not necessarily the stabilizing panacea that the New Consensus considers it to be.

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1 Introduction

The point of departure of this contribution is the Kaleckian model of growth and distribution in its simplest form with a saving and investment function, IS goods market clearing, and one dynamic mechanism that moves the economy from the short run to the long run. To be more specific and following the terminology of Lavoie (2014, p. 348), we are concerned with the (older) neo-Keynesian strand of the theory that conceives of the "normal" rate of capacity utilization as a given

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target. In this setting the problem of Harrodian instability arises, a process of cumulative causation where an excess of actual over normal utilization raises sales expectations and so drives up the firms' growth rate of fixed capital, which *via* the multiplier increases utilization, which further boosts investment, etc. On the other hand, not all theorists employing this framework view the economy as being globally unstable. Thus, they face the challenge of framing an additional mechanism that may tame the destabilizing Harrodian forces.

The problem is still an open issue. Several attempts at a solution have been provided in the literature, but so far none of them deemed generally satisfactory (see Hein et al., 2011, for a survey). The present paper focusses on the role of monetary policy (and deliberately downgrades fiscal policy).[1] Here many post-Keynesian theorists consider approaches with a conventional stabilization mechanism based on a simple interest rate inverse IS curve the effects of which are fairly straightforward in nature. These authors may even discuss a specific version, but then express their criticism and appear to discount the topic of monetary policy altogether. Our point is that this treatment cannot be the last word because it neglects other more indirect and more realistic transmission mechanisms. These elements deserve a deeper look and so will lead us to a reconsideration of the effectiveness of monetary policy *vis-à-vis* the inherent Harrodian instability.

The greater richness in our approach comes at the price of more complications in the details of the stability analysis. Seeing through the algebraic dust, one can nevertheless still draw some general conclusions. Although the effects that we will study are by no means exotic, it may, however, be anticipated that there will be no more an easy answer to the stability question. Correspondingly, monetary policy, which without doubt is a relevant feature in the real world, does have some stabilizing influence, but it is not so pervasive as the neoclassical and much of the heterodox theories tend to believe.

The paper is organized as follows. The next section explains its general perspective. Section 3 reiterates the Harrodian instability in the standard Kaleckian framework. Section 4 presents the elements that are common to the three model versions which are subsequently put forward. Here, Section 5 deals with a baseline model where the dynamics can be reduced to one differential equation in the rate of utilization. Section 6 amends this version by introducing a variable inflation climate into the Phillips curve relationship. Section 7 drops the assumption of a constant price markup and studies its dynamic adjustments over time. In fact, this

[1] A recent contribution that, starting out from Harrodian elements, studies monetary and fiscal policy in a (much) wider framework with explicit financial assets is Ryoo and Skott (2015). Clearly, there are pros and cons for 'more realistic' assumptions, so it goes without saying that simpler and more advanced models should be regarded as complementary.

may substantially qualify the previous stability results. Section 8 concludes, and an appendix contains a number of mathematical details.

2 Perspective of the model

As can be conveniently seen from the compilation by Hein et al. (2011), there are a number of approaches to cope with the problem of Harrodian instability. One of their references is Duménil and Lévy (1999) who, although they do not tackle the Harrodian knife edge itself, are included in the compilation since they proposed monetary policy as a stabilizing device. The idea of their treatment is elementary and rests on three assumptions: (i) aggregate demand is inversely related to the real rate of interest; (ii) higher than normal utilization rates are conducive to price inflation; (iii) the central bank responds to this inflation by pushing up real rates of interest. Hence a situation of overutilization increases the rate of inflation, whereupon the central bank raises the real interest rate, which in turn slows down demand and thus economic activity. Overall in this setting, there is a tendency for utilization to be brought back to normal. Several authors have pointed out that this argument is very much reminiscent of the (neoclassical) New Macroeconomic Consensus.

Another point is also emphasized. On the one hand, the economy behaves along Keynesian or Kaleckian lines in the short period, where it is demand-led and, for example, a lower propensity to save increases utilization (the paradox of thrift). On the other hand, by contrast, its long-run behaviour is consistent with the supply-led classical model of accumulation, where this lower propensity to save results in a lower rate of accumulation. In the often quoted words of Duménil and Lévy (1995, pp. 136f), "while it is possible to be Keynesian in the short term, one is required to be classical in the long term".

Many post-Keynesians are critical of this methodological point of view.[2] In addition to expressing their dislike, they argue that it unduly neglects several demand-led features of steady state positions. So the normal rate of utilization (or possibly the NAIRU in some sort of Phillips curve) may become endogenous over time by adjusting in the direction of a realized trend rate. Or the "natural" rate of growth adjusts to current growth rates through changes in the rates of technical progress, or through changes in the workforce by immigration or labour participation. Nonetheless, a modelling of these ideas need not necessarily contradict the New Consensus; it may rather complement it.[3] Besides perhaps a battle over stylish words to stand

[2] A nice example is Lavoie (2010, p. 146), who describes the disappearance of the paradox of thrift in the long run in Duménil and Lévy's conception and then concludes, "we are back to the dismal science".

[3] See Lavoie (2006) for an elementary elaboration.

out against the neoclassical body of thought (such as the phrases of the paradoxes of costs and thrift), a main motive for accentuating (some of) the aforementioned points is to maintain a perspective for macroeconomic policies that can provide an alternative to the received wisdom of mainstream economics (cf. Lavoie and Kriesler, 2007, p. 595).

These issues are, however, not the concern of the present paper. We rather wish to relate monetary policy explicitly to the Harrodian instability problem, which is something that so far has submerged in the critical discussions of the New Consensus as a whole. Accordingly, we wish to put the possibly stabilizing effects of monetary policy under closer scrutiny, because they are arguably less obvious than the short sketch above suggests. In the meantime, we consider it expedient to leave the conceptual features regarding the long-run equilibrium positions aside.

Our contribution extends the common presentation of the New Consensus, or deviates from it, in several respects. To begin with, more often than not monetary policy is modelled as a rule where the *changes* in the real interest rate are a function of the utilization gap or/and the inflation gap or/and the changes in inflation ('gap' meaning the deviation of the current rates from some benchmark value).[4] One possible reason for this priority is that it makes it easier for the central bank to achieve its target rate of inflation in a steady state (see Kriesler and Lavoie, 2007, p. 389). Another reason is that these rules need not invoke a natural rate of interest, which according to post-Keynesian monetary theory is something that does not exist (Lima and Setterfield, 2008, p. 447).

Unfortunately, this kind of literature takes no behavioural perspective. So it does not seriously discuss how reasonable an interest rate function without an anchor would be in the end. Can the central bank really afford to ignore the current level of the nominal rate of interest completely? Nor does this literature attempt to give an empirical justification. We therefore return to the standard Taylor rule, which is elsewhere a well-established design and even in its simplest version makes better economic and empirical sense. Regarding the possibly controversial issue of the central bank's estimates of the underlying "natural" real rate of interest, we may suggest to view such a benchmark as a social construction.[5]

A second point is the main channel of monetary policy in the New Consensus, where aggregate demand is unanimously assumed to be a negative function of the real interest rate. As a consequence, any change in the latter has an immediate

[4] Some examples are Lavoie (2003, 2010), Setterfield (2004, 2005), Kriesler and Lavoie (2007).

[5] Lima and Setterfield (2008, p. 454). The authors add that this is a subjective target "masquerading in the minds of central bankers as objective features of the economy" and refer to Smithin (2004) for further discussion. Generally, we do not wish to deny that there might be reasons to deviate from a Taylor rule approach or to extend it. What we wish to criticize, however, is a complete lack of a conceptual discussion in this respect.

impact on economic activity. Apart from the empirical question of a significant interest sensitivity of demand, this specification entirely disregards the acknowledged fact that the effects of monetary policy take a considerable amount of time to work out. We seek to take this feature into account by building in a certain delay. Specifically, instead upon the real interest rate, investment demand of the firms is supposed to depend on a sentiment variable on which the interest rate acts. Furthermore, the interest rate influences not the level but the *rate of change* of this sentiment. Hence some time will elapse until a (hypothetical) sudden change of the interest rate would achieve a noteworthy change in the level of investment.

Third, the sentiment changes are not dependent on the real interest rate alone, which would seem too mechanistic a relationship. After all, a firm could well be satisfied with a one-percent increase in its real interest costs if at the same time its profits rose by two per cent. We therefore stipulate that the firms' sentiment reacts to the difference between the rate of profit and the real interest rate. If our modelling approach were still assigned to the "New Consensus", it is particularly because of these three points and their more indirect interest rate effects that it would at least have to be considered a modified, or moderate, version of this theoretical view.

In a fourth point it should be mentioned that discussions of the New Consensus, even if it appears in some post-Keynesian variety, usually neglect income distribution. We will not only maintain the distinction between rentiers and workers but we will also present a version where the firms' markup on unit labour cost changes in an endogenous way, so that here also the profit share will vary over time. In detail, it will assumed that, *inter alia*, the markup reacts to the real interest rate. As indicated by Hein et al. (2011, p. 601) in their critique of Duménil & Lévy and the New Consensus in general, such an extension provides another channel of monetary policy.

Finally as our fifth point, we take up a related remark by Hein et al. (2011, p. 601) in this context where they point out that the "normal" rate of utilization will not remain unaffected by the monetary policy interventions. This observation induces us to specify a further influence of the real interest costs on the rate of *desired* utilization.

While all of these features are introduced with the ambition to give the model a flavour of a bit more 'realism', the concrete specifications will be highly stylized. Mathematically, the dynamic models that we construct from them can thus be kept to a limit of just two dimensions (in a continuous-time formulation). Nevertheless, as it has already been announced in the Introduction, the price for the greater modelling richness is that it no longer admits an easy answer to the stability issue. This means that entering the stability conditions are composite coefficients that can be somewhat involved. It will be seen, however, that the contribution of the single effects can still be clearly identified and so the conditions can be given a meaningful economic interpretation.[6]

3 The point of departure

Let us start out from the problem of Harrodian instability as it is laid out by, for example, Hein et al. (2011, Section 2). To keep things simple, it is considered for a closed one-good economy without taxation, government spending and capital depreciation. Rather than follow the practice of referring to capacity utilization as a measure of economic activity, it simplifies the notation if we directly work with the output-capital ratio u in this respect, which is the utilization rate of the capital stock in place. For short, we call it 'utilization', too. Labour is supposed to be in perfectly elastic supply, which saves us one state variable as we do not need to keep track of the employment rate. The Kaleckian IS part of the model is thus constituted by the following four basic equations:

$$
\begin{aligned}
r &= hu && (1)\\
g^s &= sr && (2)\\
g^i &= a + \gamma(u - u_d) && (3)\\
g^i &= g^s && (4)
\end{aligned}
$$

In the first equation, r is the rate of profit and h the share of profits in total income, which until Section 7 is supposed to be fixed. The function g^s in (2) represents the saving in the economy normalized by the (replacement value of the) capital stock: workers consume all of their wages and s is the capitalists' propensity to save out of their profits.

The third equation specifies the investment function, i.e. the planned growth rate of the capital stock. Within the short period it is based on a trend rate of growth a, or on the rate at which sales are expected to grow on average in the near future. Frequently, this yardstick is also introduced as the firms' "animal spirits".[7] More demurely, we may refer to a as the general business *sentiment*, a term that still preserves the psychological and somewhat diffuse character of this type of expectations. The second component of investment in (3) says that in the presence of overutilization, when actual utilization u exceeds a desired rate of utilization u_d, the firms seek to reduce this gap relatively fast by increasing the capital stock at a higher rate than a; and correspondingly for underutilization. For the moment being, u_d is treated as an exogenously given constant.

[6] Some of the technical details of how we arrive at these results are a bit awkward and will therefore be relegated to the appendix.

[7] For example, in Hein et al. (2012, p. 157) or Skott and Zipperer (2012, p. 283)

Equation (4) postulates the temporary IS equilibrium, where market clearing is brought about by quantity variations. Utilization is then given by[8]

$$u \;=\; u(a) \;=\; \frac{a - \gamma u_d}{sh - \gamma} \tag{5}$$

Stability of the underlying ultra short-run quantity adjustment process requires the denominator to be positive, that is, investment must not be too sensitive to changes in utilization. This is the so-called Keynesian stability condition, which we will not call into question. Regarding the numerator it will be noted that in a long-run equilibrium (identified by variables with a superscript 'o'), where firms operate at desired utilization $u = u^o = u_d$, the trend growth rate will be $a = a^o = g^i = g^s = shu_d$. Hence in fact $u(a^o) = (sh-\gamma)u_d/(sh-\gamma) = u_d$, and $u(a) > 0$ for all a not too much smaller than a^o.

The typical discussion of the economy (1) – (5) begins with such a steady state, where $u = u(a^o) = u_d$. Here an exogenous shift in the saving propensity s or the profit share h is hypothesized. This causes u to deviate from u_d and gives rise to the celebrated paradoxes of thrift and of costs: as it is easily seen from (5), a decrease in the saving propensity s or a rise in real wages (thus decreasing the profit share h) leads to an immediate increase in utilization. The argument put forward in this situation (at least by neo-Ricardian, Sraffian or Marxist authors, as they are often categorized) is that the firms will not accept such an over- or underutilization for too long. Given the assumption of an invariable desired utilization u_d, they conclude that it is their trend growth rate a that has been too pessimistic. Correspondingly, a is assumed to become a dynamic variable that rises (falls) if u is above (below) the desired level. Denoting the speed of these adjustments by $\eta_u > 0$, we have,[9]

$$\dot{a} \;=\; \eta_u \,[\, u(a) - u_d \,] \tag{6}$$

The process can only come to a halt if desired utilization is achieved, which as has just been seen is the case for $a = a^o$. If we now conduct the aforementioned experiment and decrease the propensity to save s, the immediate reaction according to (5) is a rise in utilization above u_d. This makes firms more optimistic regarding their future sales and they adjust a upwards. With s staying put at its new level, this increases utilization further, which in turn increases the sentiment a, etc. (although the steady state value of a^o has fallen). In other words, once the economy is perturbed from its steady state growth path, it is strongly diverging in a process of cumulative causation; the formal mathematical argument being $du/da > 0$ in (6).

[8] Since no other situations than IS are considered, we abstain from earmarking these values by an asterisk or a subscript like 'IS'.

[9] A dot above a dynamic variable x designates its derivative with respect to time, $\dot{x} = dx/dt$.

Equations (1) – (6) are a particularly simple way to formalize the Harrodian instability problem and prepare the ground for a rigorous discussion on how to overcome it.

4 The common interest rate effects

We are now going to incorporate effects into the basic setting of eqs (1) – (6) that arise from the real interest rate. At the beginning, we reconsider the concept of desired utilization. Treating u_d as a constant can be misleading if one wants to examine the stabilizing potential of monetary policy. Standard motivations for firms to maintain excess capacity imply that the desired utilization rate depends on the cost of finance (Skott, 1989). For example, if the motivation is to deter entry by rival firms, then the need to let part of the capital stock idle depends on the cost of entering the market, which in turn depends on the cost of finance; if the motivation derives from the volatility of demand and the ability to take advantage of a sudden increase in sales, this potential benefit will have to be balanced against the cost of maintaining excess capacity. With a rising interest burden firms may therefore desire to produce the same output with less capital, so that u_d increases.[10] This idea takes up the fifth point mentioned in Section 2.

Let i be the nominal interest rate and π the rate of inflation. The firms assess the current value of the real rate of interest $(i-\pi)$ against the benchmark of a "normal" real interest rate $(i^\star-\pi^\star)$, where the rates $i^\star$ and $\pi^\star$ can be thought of as the targets of the central bank. Assuming that these targets are shared by the firms as a natural reference may mean that monetary policy is sufficiently credible to them (although this is not a necessary interpretation). To simplify the analysis we proxy the firms' cost of finance by the real interest rate and use a linear specification for desired utilization:[11]

$$u_d \;=\; u_f^\star \;+\; \beta_d\,[\,(i-\pi) - (i^\star - \pi^\star)\,] \tag{7}$$

The coefficient β_d is nonnegative, as all other parameters in the following if nothing else is stated. If one holds the view that the variations in u_d are weaker than those of

[10] This is not the only channel through which monetary policy might take effect. Thus, Hein et al. (2012, p. 160) by invoking a simple conflicting-claims model of inflation derive a negative dependence of their normal rate of utilization upon the real interest rate. Our inflation theory below, however, will follow a different route.

[11] Without the linearity assumption there may be no explicit solution for the IS equilibrium below. The stability analysis will nevertheless be exactly the same when for the partial derivatives of IS utilization we make use of the Implicit Function Theorem. The details of this procedure are provided in the appendix.

utilization itself, the coefficient should be limited in scale.[12] A reasonable upper-bound might then seem to be around one, which will play a certain role in the stability analysis later on. The symbol $u_f^\star$ means that the firms have an idea about what their utilization should be under a 'normal' cost of finance, when inflation and the real interest rate are on target. Correspondingly, $u_f^\star$ can be referred to as the firms' perception of 'normal' utilization. We append a subscript 'f' in the notation ('f' for firms) since in general the central bank might work with a different level for this concept in its policy.

It will be noted that by substituting u_d from (7) in the IS equilibrium (5), a negative effect of the real interest rate on economic activity is obtained. It is, however, of a different nature from the conventional arguments (if these are ever made explicit and discussed, it should be added). Moreover, given the remark on β_d and that because of the Keynesian stability condition the coefficient γ must not be very large, this effect will probably be rather weak. So not too much may depend on it. Besides, the analysis will also allow for its complete absence, when $\beta_d = 0$.

Another and more dynamic effect of the interest rate originates with its role as an alternative return to the profit rate. In this respect we turn to the investment decisions of the firms and follow up on eq. (6) for the sentiment variable a. While its positive response to the utilization gap $(u - u_d)$ is maintained, we now additionally take into account that a possible optimism of firms from a current overutilization should be reasonably backed up by a sufficient profitability of fixed investment. Specifically, the firms (or their owners) will compare the rate of profit with the returns from the safer investment in government bonds. Accordingly they compare the difference between r and the real rate of interest $(i-\pi)$ to a desired positive return differential $\rho^\star$ that reflects what they think would be a decent risk premium. Accordingly, the benchmark growth rate a is increased if $r - (i-\pi)$ exceeds $\rho^\star$, and decreased if it falls short of $\rho^\star$.[13] Taken together, with two nonnegative reaction coefficients η_u and η_ρ we hypothesize:

$$\dot{a} \quad = \quad \eta_u\,(u - u_d) \;+\; \eta_\rho\,[\,r - (i-\pi) - \rho^\star\,] \tag{8}$$

Equation (8) specifies the second and third point mentioned in Section 2, saying that it takes some time until changes in the interest rate will have a sizeable effect on investment (and thus utilization), and that relevant for the firms is not the interest rate itself but the return differential.

[12] Figure 2 in Schoder (2012, p. 9) may be taken as a support of this view (where in detail, however, his 'utilization' is capacity and not capital utilization).

[13] To be exact, the interest rate entering (7) will be a loan rate and the one in the following equation (8) a bond rate. To avoid the issue of modelling a term structure of interest rates, the two may be assumed to move in step. Thus the same symbol can be used in (7) and (8) because it is only their deviations from a (suitably scaled) reference rate $i^\star$ or $\rho^\star$ that matters.

It will also be observed that the utilization gap $(u-u_d)$ takes several dynamic effects. First, it directly enters the investment decisions in (3), and second, it shifts the entire investment function over time *via* (8), an effect that would continue if (hypothetically) the gap persisted. It might be argued that these specifications in the firms' investment behaviour overemphasize the role of utilization, or violate the principle of a parsimonious modelling. In this case either γ in the investment function (3) could be set equal to zero or $(u-u_d)$ could be eliminated from (8) by putting $\eta_u = 0$; both cases will be admissible in the stability analysis.

If the utilization gap and therefore the Harrodian instability mechanism is maintained in the adjustment equation (8) then, third, there is an indirect effect of u *via* the profit rate $r = hu$ in its second term (in addition to the direct effect of u in the first term). This also shows that in the presence of a fixed real interest rate the second feedback in the sentiment adjustments would only reinforce the Harrodian mechanism.

A fourth point is that if on the other hand, as it will be formulated in a moment, the interest rate itself reacts to changes in utilization, then *via* the return differential in (8) we have a further indirect effect of u on the business sentiment. A similar argument may apply to the inflation rate π, if it is supposed to change in line with economic activity. The significance of the combination of these two effects in the real rate of interest is that they may provide a certain counterbalance to the Harrodian forces and thus contribute to a stabilization of the economy.

Apart from these stability issues, it is remarkable that with two factors determining the change in the business sentiment it is no longer automatically guaranteed that $u = u_d$ in a steady state position, as in Section 3. Consider, for example, an overutilization $u > u_d$, which according to eq. (8) would motivate the firms to increase a. At the same time the return differential may be less than the required risk premium, which would dampen the firms' optimism. Depending on the precise values of the coefficients η_u and η_ρ a situation may arise where the firms are not necessarily happy with the current value for the trend growth rate, but they experience a tension between two motivations that point in opposite directions and just happen to neutralize each other. It hinges on the other parts of the model whether such a situation may even reproduce itself over time and give rise to a long-run equilibrium. This topic will be further discussed in Section 5.

With these remarks we turn to the determination of the interest rate. A standard device is an interest rate reaction function adopted by the central bank. While over the last twenty years it has become a central element in the New-Keynesian macroeconomic school, it certainly should not be left to these rational expectations models alone.[14] The concept first says that the central bank does care about the *level* of the nominal rate of interest, in contrast to the literature emphasizing the *changes* in the interest rate cited in Section 2. It therefore uses a rate $i^\star$ as an anchor. Secondly, the central bank seeks to achieve a target rate of inflation $\pi^\star$ and a normal output-capital ratio.[15] In distinction from the firms' utilization target, we denote the central bank's target by $u^\star_{cb}$. The latter may be different from $u^\star_f$ when the central bank does not have the same precise knowledge about production costs and technology as the firms. In addition, it also has a longer time perspective, so that it may deliberately neglect the effect of the current real interest rate on desired utilization u_d.

Concerning the precise specification of monetary policy it is convenient for us to work with the original Taylor rule (Taylor, 1993, p. 202), which means the interest rate is a direct function of the utilization and inflation gaps. Accordingly, the central bank raises the interest rate above $i^\star$ if it observes utilization or inflation above their targets ($u > u^\star_{cb}$ or $\pi > \pi^\star$, respectively), and likewise in the other direction.[16] With positive monetary policy coefficients μ_u and μ_π, the interest rate reaction function thus reads,

$$i \quad = \quad i^\star \,+\, \mu_u\,(u - u^\star_{cb}) \,+\, \mu_\pi\,(\pi - \pi^\star) \tag{9}$$

Certainly, if a "natural" real rate of interest is regarded as the central bank's primary goal, $i^\star$ is the sum of this rate and target inflation. We assume that the central bank obeys the so-called Taylor principle, according to which it increases the interest rate not less than one-to-one in response to a rise in inflation. Hence $\mu_\pi \geq 1$.[17]

Of course, the interest rate under the control of the central bank is a short-term rate. Again, however, as already indicated in footnote 13, we may assume that the loan and bond rates from above result from a simple markup procedure. The fact

[14] Chapters 8 and 9 in the book by Chiarella et al. (2005) can be viewed as an extensive (also numerical) discussion of the effects of a Taylor rule in an elaborated heterodox modelling framework (in contrast to the traditional LM setting underlying the chapters before).

[15] Because we have no normative interest in monetary policy, we abstain from further arguments in the policy function.

[16] Since we will only study local stability, we need not be concerned about the zero lower-bound for interest rates. One hardly needs an extensive theoretical discussion in order to conclude that this restriction would make conventional monetary policy ineffective under severe deflationary pressure or/and in a deep recession, when also the firms' animal spirits may have collapsed.

[17] Once in a while, a little tribute may be paid to the ancestors of the basic idea of inflation targeting. After all, it was around 100 years before Taylor that Wicksell built a positive effect of price inflation on the interest rate into his cumulative process determining inflation, and Thornton was saying much the same thing almost another century before Wicksell. Furthermore, by rewriting (9) as $i - \pi = i^\star - \pi^\star + \mu_u(u - u^\star_{cb}) + (\mu_\pi - 1)(\pi - \pi^\star)$ it is also directly seen that $(i^\star - \pi^\star)$ is, in Wicksellian terms, the central bank's estimate of the "natural" real rate of interest.

that for all three interest rates only the deviations from some constant benchmark rates matters saves us from introducing an extra notation and we can work with one symbol 'i' in the three functional relationships (7), (8), (9).

To close the model, we still need a building block that determines the rate of inflation. Here three different versions will be proposed that give rise to three model variants. They will be the subject of the following sections.

5 The dynamic baseline model

5.1 A simple inflation theory

The New Macroeconomic Consensus holds the view that rising economic activity intensifies the pressure on prices and so tends to increase inflation. In other words, it subscribes to some sort of Phillips curve relationship. While economic activity will again be measured by the firms' utilization gap, the interest costs determining desired utilization may be neglected in this context, so that u is related to $u_f^\star$ rather than u_d.[18] On the other hand, expectations about future inflation will have to be given a role, too. In the present section we adopt the simplest assumption for an inflationary environment, which is a constant rate of expected inflation equal to the central bank's target rate $\pi^\star$ in the Taylor rule. The economic meaning of this scenario is that monetary policy enjoys maximal credibility (an assumption that will be relaxed in the next section). With a nonnegative slope parameter β_w (the motivation for the index 'w' becomes clear in a moment), the following price Phillips curve is stipulated:

$$\pi \;=\; \pi^\star + \beta_w (u - u_f^\star) \tag{10}$$

There are several ways to interpret this relationship. With a view to the extension of the model in Section 7 below, it may be derived from a wage Phillips curve and a constant markup in the firms' pricing policy. Let w be the nominal wage rate, z labour productivity, $\hat{w} = \dot{w}/w$ and $\hat{z} = \dot{z}/z$ their growth rates, and for simplicity proxy an employment gap with $(u - u_f^\star)$. The wage adjustments then read,[19]

$$\hat{w} \;=\; \pi^\star + \hat{z} + \beta_w (u - u_f^\star) \tag{11}$$

[18] The basic conclusions would nevertheless not be affected by this simplification.

[19] Our approach does not rule out that wages respond to an employment gap where in a post-Keynesian fashion the stable-inflation employment rate $e^\star$ is derived from conflicting claims over distribution. This could be done, and refined, along similar lines to Hein and Stockhammer (2010, pp. 323ff), for example. In general, however, such an $e^\star$ may not be compatible with our present $u_f^\star$ in (11), which would imply that capital utilization in a steady state will be different from $u_f^\star$. Nevertheless, this phenomenon is independent of any assumptions about monetary policy.

If p denotes the price level and τ the constant markup rate on unit labour cost, the price side is represented by the equation

$$p \;=\; (1+\tau)\,w/z \tag{12}$$

(the markup will become an endogenous variable in Section 7). Hence price inflation is $\pi = \hat{p} = \dot{\tau}/(1+\tau) + \hat{w} - \hat{z}$, which is nothing else than eq. (10).

As one of the two main critiques of Duménil & Lévy, Lavoie (2014, p. 398) cites empirical evidence that over a medium range the Phillips curve exhibits a horizontal segment. For this reason β_w is here also allowed to be zero or very low. In the second part of his remark Lavoie adds (line −4 on p. 398) that "hence [...] the monetary authorities will not feel the need to impose more restrictive policies" during these episodes. This idea could well be captured by setting μ_π in (9) equal to one, whereas the central bank may continue to pursue an active policy with respect to economic activity, so that μ_u is strictly positive.[20]

The advantage of the constant target inflation in (10) is that it allows us to reduce the dynamics of the sentiment variable a to just one differential equation. In a first step of the analysis, we can make use of the Taylor rule (9) and the price Phillips curve (10) and express the IS rate of capital utilization as an increasing function of a (see eq. (A4) in the appendix for the details):

$$u \;=\; u(a)\,, \qquad \text{where } \frac{du}{da} \;=\; \frac{1}{\sigma} := \frac{1}{sh \,-\, \gamma \,+\, \beta_d\,\gamma[\mu_u + (\mu_\pi - 1)\beta_w]} \tag{13}$$

The denominator is positive by virtue of the Keynesian stability condition mentioned above. The influence of the central bank can be seen in the two policy coefficients μ_π (in the presence of $\beta_w > 0$) and μ_u, provided desired utilization reacts to the interest rate ($\beta_d > 0$) and the investment function incorporates the changes in utilization ($\gamma > 0$). Under these circumstances, strong reactions in monetary policy could somewhat reduce the sentiment multiplier, though this effect should not be overstated.

By straightforward substitution the real interest rate and thus also desired utilization u_d can be written as functions of u. The same is possible for the return

[20] Setting $\mu_\pi = 1$, however, ignores Taylor's (1993, p. 202) widely acknowledged statement that, even without going into econometric details, empirical observations suggest a benchmark around $\mu_\pi = 1.50$. As far as Lavoie's first critique of Duménil & Lévy is concerned, we will turn to it further below. As a third criticism Lavoie briefly mentions that "when interest payments to households are taken into consideration [...] it is not clear that higher interest rates will have the expected restrictive impact on the economy" (Lavoie, 2014, p. 399). Such a feature would open up a completely new discussion, where this temporary equilibrium effect is only one among several others. Besides, one should then also include the financing of fixed investment into a model, where higher debt services would cut into the firms' profits. Before addressing the possible effects in such an augmented framework, there is already enough to do in the present setting.

differential $\rho := r - (i - \pi) = hu - (i - \pi)$ in the sentiment adjustments (8) (see eqs (A1)–(A3) in the appendix). Substituting $u_d = u_d(u)$ and $\rho = \rho(u)$ in (8), we get

$$\dot{a} \quad = \quad \eta_u [u - u_d(u)] \; + \; \eta_\rho [\rho(u) - \rho^\star] \, , \qquad \text{where } \; u = u(a) \tag{14}$$

5.2 Discussion of the steady state position

Before discussing the stability of process (14) let us consider its point of rest, which constitutes a steady state of balanced growth. Firms do not necessarily produce at their desired rate of utilization in this long-run equilibrium, or it may not equal their normal utilization rate $u_f^\star$. It is easily seen that a harmonic state prevails if two consistency conditions on the target values in the economy are met (recall that the steady state values are identified by a superscript 'o').

Proposition 1

> Suppose $u_{cb}^\star = u_f^\star =: u^\star$ and $i^\star - \pi^\star = r^\star - \rho^\star$, where $r^\star = hu^\star$. Then $u^o = u_d(u^o) = u^\star$ and $\rho(u^o) = \rho^\star$ in the (unique) steady state of (14), that is, both disequilibrium gaps in (14) vanish in this equilibrium. The growth rate is determined by $g^o = a^o = sr^\star$ and the real interest rate by $i^o - \pi^o = i^\star - \pi^\star$.

The first condition says that the central bank shares the firms' perception of normal utilization. According to the second condition, it sets its targets $i^\star$ and $\pi^\star$ such that investors with a required risk premium $\rho^\star$ would be indifferent between the rates of return of government bonds and real capital in the equilibrium state. Note that $(r^\star - \rho^\star)$ could be viewed as the economy's natural real rate of interest, and the condition says that the central bank has succeeded in estimating it correctly.[21]

The steady state continues to be uniquely determined if the central bank does not meet the consistency conditions in the proposition (since (14) is a linear process in a, and if we ignore the meaningless possibility that $d\dot{a}/du = 0$). However, except for a fluke, the two gap terms in (14) would be nonzero then. One of them pushes the sentiment variable upwards, the other downwards, and these opposite forces just balance. This also means that here a *ceteris paribus* change in the reaction coefficients η_u and/or η_ρ would cause a shift in the equilibrium values.

[21] Lavoie (2006, Section 3.2) discusses the determination of a natural real rate of interest j_n in a simple three-equation model (without profit rate) where the real interest rate impacts on aggregate demand directly. Within such a framework, j_n additionally depends on the corresponding slope parameter. It may be argued that determining j_n as $(r^\star - \rho^\star)$ without such a possibly somewhat volatile reaction parameter is more appropriate for a concept with the attribute "natural".

A possible deviation of actual from desired utilization in an equilibrium is the first of Lavoie's (2014, p. 398) "two main critiques" to the mechanism proposed by Duménil & Lévy. To substantiate his assessment he refers to a small model that is quite special in our opinion. We believe the reason that causes the deviation in our model is of more fundamental importance, which is the existence of two dynamic feedback loops acting on one variable. Such a phenomenon is, however, not limited to the present modelling of monetary policy and its transmission channels; it can easily show up in any model with only a moderately richer structure.

It may additionally be noted that a state where not all variables are on target need not infinitely persist. Correspondingly, such a stationary point of process (14) could be viewed as a medium-run equilibrium. For example, the central bank may seek to counteract to the deviation of $(i^o - \pi^o)$ from its policy target $(i^\star - \pi^\star)$ by adjusting the latter until the two are equal. In the lucid case $u^\star_{cb} = u^\star_f = u^\star$, this also goes along with normal utilization $u^o = u^\star$ (the reason is eq. (A1) in the appendix). In greater detail, an obvious reaction pattern for the central bank would be that if it observes or infers that, say, $(i^\star - \pi^\star) < (i^o - \pi^o)$, it revises the target upward—expecting that this reduces rather than increases the difference between these two rates. It is shown in the appendix that such a presumption is indeed warranted if (and only if) Assumption 1 in the next subsection is satisfied.[22]

5.3 Stability of the steady state

The following Assumption 1 is of crucial importance in the stability analysis of the sentiment dynamics (14).

Assumption 1

$$\begin{aligned} \mu_{u\pi} &> h\,, \quad \text{where } \mu_{u\pi} := \mu_u + (\mu_\pi - 1)\,\beta_w\,, \quad \text{and furthermore} \\ (\mu_{u\pi} - h)\,\eta_\rho &> (1 - \beta_d\,\mu_{u\pi})\,\eta_u \end{aligned}$$

To assess the first inequality $\mu_{u\pi} > h$, note that Taylor's (1993, p. 202) famous benchmark value for μ_π in his policy rule is $\mu_\pi = 1.50 > 1$, while for the coefficient on an output gap (i.e. the percentage deviations of output from its trend or some concept of potential output) he proposes 0.50. In the present setting, the counterpart of the latter measure of economic activity is $(u - u^\star_{cb})/u^\star_{cb}$. Hence the coefficient on $(u - u^\star_{cb})$ in the policy function (9) would be given by $\mu_u = 0.50/u^\star_{cb}$.

[22] In the alternative case of $u^\star_{cb} \neq u^\star_f$, the central bank would at least be able to achieve $(i^\star - \pi^\star) = (i^o - \pi^o)$, while u^o would be between the two benchmarks $u^\star_{cb}$ and $u^\star_f$ (according to (A1)).

Moreover, for the output-capital ratio $u^\star_{cb}$ we can work with a value less than unity; for example, Hein and Schoder (2011, p. 702) report a time average of $u = 0.833$ for US data over the period 1960 – 2007, and Chiarella et al. (2005, p. 85) obtain values between 0.65 and 0.90 in the post-war period. On the other hand, a rough-and-ready guess of the profit share is $h = 0.30$, while Hein and Schoder (2011, p. 702) compute an even lower time average of $h = 0.22$. Taken together, the first inequality in Assumption 1 on the central bank's responsiveness can be safely taken for granted, even in the presence of a horizontal Phillips curve ($\beta_w = 0$). Therefore, when reference is made to a situation where Assumption 1 is violated, only the reverse inequality sign in the second part will be meant.

Let us then consider the dynamic effects in process (14). Clearly, if $\beta_d = 0$ the utilization gap in (14) gives rise to the same destabilizing Harrodian mechanism as in Section 3. Since differentiation of the gap with respect to u yields $(1 - \beta_d\,\mu_{u\pi})$ and $du/da > 0$ from (13), this positive feedback will generally be weakened when $\beta_d > 0$. It could even be twisted to the opposite if β_d is sufficiently high. However, we do not want to make stability dependent on such strong reactions in desired utilization (7) and take $\beta_d < 1/\mu_{u\pi}$ as the normal case, i.e., the Harrodian feedback is still active.

With this evaluation we face the question of whether the return differential $\rho = r - (i - \pi)$ is another channel through which monetary policy could exert a stabilizing effect. From eq. (A3) in the appendix it can be seen that the derivative of ρ with respect to u is $h - \mu_{u\pi}$. Hence the first part of Assumption 1 is equivalent to saying that the return differential produces a negative feedback of a onto itself in the sentiment adjustments (as $d\rho/du < 0$ and $du/da > 0$). This effect can furthermore dominate the Harrodian mechanism and thus bring about stability if the reaction coefficient η_ρ is sufficiently high relatively to the coefficient η_u. This is what the second inequality in Assumption 1 amounts to, that is, it is equivalent to a negative derivative of the entire right-hand side of (14) with respect to u. In sum, the following stability result obtains.

Proposition 2

An equilibrium point of the sentiment dynamics (14), which is unique, is globally stable if, and only if, Assumption 1 holds true.

A back-of-the-envelop calculation can serve to assess the second inequality in Assumption 1. Let us borrow $u^\star_{cb} = u^\star_f = 0.833$ from the empirical remarks above. In a fairly conservative manner we may moreover assume $\beta_d = 0.50$ and concede a horizontal Phillips curve, $\beta_w = 0$. Hence, $\mu_{u\pi} = \mu_u = 0.50/0.833 = 0.60$ and $1 - \beta_d\,\mu_{u\pi} = 0.70 > 0$ (positive values of β_w would not change this figure very much). This makes sure the Harrodian forces have not disappeared.

With a profit share $h = 0.30$ we have $\mu_{u\pi} - h = 0.30$. The inequality implying stability then reads $\eta_\rho > (0.70/0.30)\,\eta_u = 2.33\,\eta_u$. If it is taken into account that the typical deviations of the return differential from normal will be somewhat less than those of the utilization gap, the inequality can be satisfied and stability can prevail if in the sentiment adjustments the firms attach a moderately stronger weight to their profitability motive than to their utilization motive. A positive slope in the Phillips curve would further broaden the stability prospects. Nonetheless, the stability condition in Assumption 1 would be an empirical issue in the end. *A priori* and for the moment being, however, it does not appear too exotic, either.

A more specific question is whether monetary policy, at least in principle, could stabilize an originally unstable economy by sufficiently strong reactions in the Taylor rule. The answer is an unqualified 'yes' for the responses to the utilization rate, because an increase in μ_u raises the left-hand side and lowers the right-hand side in the second inequality in Assumption 1. As fluctuations in the inflation gap are typically somewhat smaller than in the utilization gap and the slope coefficient β_w in the Phillips curve will typically be considered to be less than one, the central bank's responses to inflation are less efficient in this respect (see the coefficient $\mu_{u\pi}$). These responses even prove completely ineffective if the Phillips curve exhibits a flat segment around normal utilization $u_f^\star$ ($\beta_w = 0$). This is an interesting observation as the relatively minor role of inflation targeting as opposed to the targeting of economic activity is not exactly a conventional economic wisdom.

Let us conclude this section with the observation that the linearity in process (14) allows us to have another look at an equation that Hein et al. (2011, p. 599) and Lavoie (2014, pp. 396f) put forward as a succinct characterization of the above cited model by Duménil and Lévy (1999). In the present notation, their equation reads,

$$\dot{a} \;=\; -\chi\,(u - u_d)\,, \qquad \chi > 0 \tag{15}$$

Lavoie derives the coefficient χ from a price Phillips curve $\pi = \chi_1(u - u_d)$, an accelerationist interest rate reaction function $d(i-\pi)/dt = \chi_2\,\pi$, and a direct negative feedback on the (level of the) animal spirits, $a = g^o - \gamma_i(i - \pi)$ (and not on their rate of change). In this way, $\chi = \gamma_i\,\chi_1\,\chi_2 > 0$.

Within our framework and supposing $u_{cb}^\star = u_f^\star = u^\star$, eq. (14) can be reformulated as

$$\begin{aligned} \dot{a} \;&=\; -\chi\,(u - u^\star) \;+\; [hu^\star - (i^\star - \pi^\star) - \rho^\star]\,\eta_\rho \\ \chi \;&:=\; (\mu_{u\pi} - h)\,\eta_\rho \;-\; (1 - \beta_d\,\mu_{u\pi})\,\eta_u \end{aligned} \tag{16}$$

Equation (15) is here re-established if first the consistency condition on the central bank's target rates in Proposition 1 holds, which results in a zero square bracket;

and if second Assumption 1 is satisfied, which is equivalent to $\chi > 0$. In contrast to Lavoie's story, however, as discussed above, our standard specification of the Taylor rule seems more reasonable, the interest rate feedbacks are of a more indirect nature, and they may even work with a horizontal Phillips curve. On the other hand, stability is not automatically guaranteed but dependent on a condition that may or may not be considered acceptable.

6 Introducing a variable inflation climate

The baseline model has assumed a maximal confidence in the central bank's monetary policy, in the sense that regarding future inflation the wage bargaining in (11) was directly based on the target rate of inflation $\pi^\star$. It might be suspected that this is not an innocent simplification but unduly biases the results towards stability. The present section therefore relaxes the assumption, which begins by replacing the constant $\pi^\star$ in (11) and (10) with a variable that we like to call an "inflation climate".[23] Denoting it by π^c, the price Phillips curve (10) becomes

$$\pi \;=\; \pi^c + \beta_w (u - u_f^\star) \tag{17}$$

The climate is gradually updated in the direction of a moving target. A straightforward specification of such an inflation benchmark is a weighted average of just two inflation rates: the inflation π currently observed and the central bank's target $\pi^\star$.[24] Letting χ be the (constant) weighting factor for the latter[25] ($0 \le \chi \le 1$) and α_π the speed at which the adjustments take place, we have

$$\dot{\pi}^c \;=\; \alpha_\pi [\, \chi \pi^\star + (1-\chi)\pi \;-\; \pi^c \,] \tag{18}$$

The parameter χ indicates the relative importance that the agents attach to the inflation target *vis-à-vis* past inflation experience. That is, χ expresses the degree to which the inflation climate is anchored on the central bank's target: the higher χ the higher the agents' confidence that the central bank will soon be able to bring inflation back to normal. The coefficient can thus serve as a measure of the central bank's credibility. In fact, in the special case of maximal credibility, $\chi = 1$, we would be back to the baseline model with eq. (10), since π^c in (18) would then

[23] This notion is slightly different from the usual "expected inflation" in so-called expectations-augmented Phillips curves. For an extensive discussion of the concept, see Franke (2007).

[24] This idea can be traced back (at least) to Groth (1988, p. 254). It has later been used in much of the work of Chiarella, Flaschel and others; see, e.g., Chiarella et al. (2005, from Chapter 2 onwards).

[25] Since no more reference will be made to eqs (15) or (16) above, symbol χ should cause no confusion.

independently of the rest of the economy converge to $\pi^\star$. The other polar case of minimal credibility, $\chi = 0$, corresponds to what some decades ago was often employed in monetary economics as "adaptive expectations".[26]

The other parts of the model are taken over unaltered. The difference from the baseline model is that now, with $\chi < 1$, we have π^c as a second dynamic state variable. In other respects the analysis proceeds in the same steps as before. Utilization in the IS equilibrium is here represented by a function

$$u \;=\; u(a,\pi^c)\,, \qquad \text{where } \frac{\partial u}{\partial a} \;=\; \frac{1}{\sigma}\,, \quad \frac{\partial u}{\partial \pi^c} \;=\; \frac{-\beta_d\,\gamma\,(\mu_\pi - 1)}{\sigma} \tag{19}$$

and $\sigma > 0$ was defined in (13). The reason for the negative effect of the inflation climate on u is that, first, π^c increases inflation in the Phillips curve, upon which the interest rate in the monetary policy rule increases (more than one-to-one), which in turn raises desired utilization u_d. As a result, investment in (3) is cut down and the utilization rate, *via* the multiplier $1/\sigma$, decreases. If desired utilization is independent of the interest costs ($\beta_d = 0$) or there is no direct effect of the utilization gap on investment ($\gamma = 0$), utilization would remain unaffected by the variations of π^c.

With the same reasoning as in the baseline model, desired utilization u_d and the return differential ρ can be expressed as functions of u and thus, *via* (19), of a and π^c. In this way a two-dimensional linear differential equations system in the latter two variables is obtained, which can be compactly written as:

$$\begin{aligned} \dot a \;&=\; \eta_u\,[\,u - u_d(u,\pi^c)\,] \;+\; \eta_\rho\,[\,\rho(u,\pi^c) - \rho^\star]\,, \qquad u = u(a,\pi^c) \\ \dot\pi^c \;&=\; \alpha_\pi\,\{\,\chi\pi^\star + (1-\chi)\,[\pi^c + \beta_w\,(u - u_f^\star)] \;-\; \pi^c\,\} \end{aligned} \tag{20}$$

As before, a well-behaved steady state position with no disequilibrium gaps can be shown to be established by the two consistency conditions $u^\star_{cb} = u^\star_f = u^\star$ and $i^\star - \pi^\star = r^\star - \rho^\star = hu^\star - \rho^\star$; here $u^o = u^\star$, $g^o = a^o = sr^\star$ and $\pi^{c,o} = \pi^o = \pi^\star$.

Whether or not it is well-behaved, the steady state is stable if (and only if) the (2×2) Jacobian matrix J of (20) has a negative trace and a positive determinant. From the baseline model, the entry $j_{11} = \partial\dot a/\partial a$ proves to be negative exactly if Assumption 1 is satisfied, while the other diagonal element $j_{22} = \partial\dot\pi^c/\partial\pi^c$ is negative at least if $\chi < 1$ (again, see the appendix). Assumption 1 also ensures a positive determinant, but it should be stressed that its violation need not necessarily destroy the positive sign. The precise stability result is stated in Proposition 3.

[26]They have recently experienced a certain rehabilitation as (a simplified variant of) "constant gain learning".

Proposition 3

> *Suppose* $0 \leq \chi < 1$ *and abbreviate* $\eta_{\rho u} := (\mu_{u\pi} - h)\,\eta_\rho \,-\, (1 - \beta_d\,\mu_{u\pi})$ η_u. *Assumption 1, i.e.* $\eta_{\rho u} > 0$*, is a sufficient but no longer necessary condition for stability. In detail, defining* $\widetilde{\eta}_{\rho u} := \eta_{\rho u}/(\eta_\rho + \eta_u\beta_d)$*, sufficient and necessary for stability are*
>
> $$\begin{aligned} \eta_{\rho u} &> -\alpha_\pi\,[\,\beta_d\gamma(\mu_\pi - 1)\beta_w + \chi(sh - \gamma + \beta_d\gamma\mu_u)\,] \\ \widetilde{\eta}_{\rho u} &> -(1-\chi)(\mu_\pi - 1)\beta_w\,/\,\chi \end{aligned}$$

As it is formulated, the proposition makes precise that, perhaps somewhat surprisingly, a lower credibility of the central bank (a lower value of χ than one) broadens rather than narrows the scope for stability, which technically means the composite coefficient $\eta_{\rho u}$ may violate Assumption 1 and be moderately negative, in line with the two inequalities in the proposition. Note, however, that according to the second inequality in the proposition, this case presupposes a non-horizontal Phillips curve.[27]

To see the economic reason for the greater stabilization potential, consider a situation where $\pi^c = \pi^\star$ and $u > u^\star_f$. Here current inflation π in the Phillips curve (17) exceeds the target $\pi^\star$, the climate adjustment process (18) implies $\dot{\pi}^c > 0$ when $\chi < 1$, and this in turn increases π in the Phillips curve even further. As a consequence, the real interest rate reacts more strongly than under $\pi^c \equiv \pi^\star$ in the baseline model (mathematically, this can be seen from eq. (A6) in the appendix) and so strengthens the stabilizing forces in the economy.

It may moreover be noted that the expressions in the two inequalities of Proposition 3 can be arbitrarily negative if the adjustments of the inflation climate are sufficiently fast (α_π sufficiently high) and the central bank's credibility is low enough (χ sufficiently close to zero). Interestingly, even the profitability motive would not necessarily be needed in the present model version to overcome the Harrodian instability, that is, the stability conditions in the proposition could be satisfied despite $\eta_\rho = 0$ (presupposing that, however weakly, the interest rate channel *via* desired utilization is still active: β_d possibly small but positive). While these effects may not be overrated, they show that "backward-looking" inflation expectations need not always be destabilizing.

Although our model is more limited than the policy framework developed by Ryoo and Skott (2015; RS in the following), which was mentioned in footnote 1, it still bears a certain relationship to their case of a pure monetary policy. This can

[27] Regarding the coefficient $\widetilde{\eta}_{\rho u}$, it may be observed that it decreases when η_u increases and η_ρ decreases.

be seen by freezing their two additional dynamic variables (i.e. the ratios of government bonds to capital and of capital to the labour force). As a direct analogue of our process (20), the constraint sets up two differential equations in a and π^c (RS call the latter inflation 'expectations' and denote them by π^e). The stability conditions for this reduced system are, however, tighter than ours. In particular, strictly positive reactions of desired utilization to the real interest rate are there a necessary requirement for stability ($\beta_d > 0$ in the present notation; RS's specification of u_d coincides with our formulation in (7)).[28]

The dissimilarity in the results is not so much due to RS's more elaborated consumption function but to their counterpart of our equations (3) and (8), which differs in two respects: (i) the presumably stabilizing influence of the real interest rate is there directly included in the investment function and not in the dynamic adjustments of the animal spirits; and (ii) RS only consider the cost argument of the interest rate and do not relate it to the profit rate, that is, they make no reference to a return differential.[29] An evaluation of the difference in these detailed model specifications will depend on whether or not one is willing to accept the idea that the influence of the interest rate on desired utilization is a central channel for monetary policy—given that this is a mechanism that as yet has not been rigorously discussed in the literature.

7 Introducing dynamic markup adjustments

This section takes up a criticism that Hein et al. (2001, p. 601) level at the way in which Duménil and Lévy (1999) and others incorporate the New Consensus.[30] They observe that "[h]igher (lower) real interest rates in order to adjust $u^\star$ [i.e., IS utilization] towards u_n [i.e., normal utilization] mean higher (lower) interest costs for firms. This will affect their target mark-up on unit labour costs, because the mark-up has to cover not only firms' profits but also overhead costs including interest payments." Having a theory of conflict inflation in mind, they continue, "as

[28]To verify this claim, one may consider the matrix $J(a,b,k,\pi^e)$ in RS on pp. 34f and compare the (2×2) submatrix formed by the entries j_{11}, j_{14}, j_{41}, j_{44} with our Jacobian (A8) in the appendix. Incidentally, the determinant is always positive and the trace is negative iff, in RS's notation, $\lambda - \mu\eta_1\widetilde{\rho}_3 a_2 < (\lambda\widetilde{\rho}_1 + \mu\eta_1\widetilde{\rho}_3 a_1)\,\theta$, where their θ is our β_d. Using RS's numerical values from their Section 5, one can compute that this condition amounts to $\beta_d > 0.320$, that is, the restriction on β_d is relatively mild.

[29]A consequence of this treatment is that RS's solution for the IS utilization rate involves (much) more complicated feedback effects than our eq. (19). Comparable effects are not absent in our model, they only become operative in the dynamic part.

[30]However, they do not offer or make reference to modelling attempts that would take their criticism into account.

soon as firms attempt to recover higher interest costs by means of raising the target mark-up, the distribution conflict is fuelled anew and inflation will rise again."

This reasoning indicates a possibly destabilizing effect of monetary policy. In order to inquire into this idea, we return to the convenient wage Phillips curve (11) from the baseline model but drop the assumption of a fixed markup rate τ in the firms' price setting. Instead, τ is subjected to dynamic adjustments. Certainly, the markup flexibility also induces variations of income distribution over time, as with the markup equation (12) the profit share is given by

$$h \;=\; h(\tau) \;\;=\;\; \tau/(1+\tau) \tag{21}$$

The changes in τ will therefore provide another effect on effective demand and economic activity.

Three factors are considered to which the markup may respond: the real interest costs just mentioned, the utilization gap, and as an anchor a markup rate $\tau^\star$ that the firms regard as normal. To shorten some of the formulae below we may right from the beginning assume that $u^\star_{cb} = u^\star_f = u^\star$ (the stability conditions will remain completely unaffected by this). In specifying the changes in the markup it is useful to refer to the growth rate of $(1+\tau)$,

$$\dot{\tau}/(1+\tau) \;\;=\;\; \beta_{\tau u}(u-u^\star) \;+\; \beta_{\tau i}\,[(i-\pi)-(i^\star-\pi^\star)] \;-\; \beta_{\tau\tau}(\tau-\tau^\star) \tag{22}$$

Besides $\beta_{\tau i} \geq 0$ owing to the observation by Hein et al. (2011) from above, the coefficient $\beta_{\tau\tau}$ for the autofeedback is supposed to be positive for obvious reasons. With respect to $\beta_{\tau u}$ arguments have been put forward in the literature according to which the firms may increase or decrease the markup in a boom. The stability analysis can allow for both cases and so we may leave the question of the sign of $\beta_{\tau u}$ open. A degenerate case is obtained if $\beta_{\tau u} = \beta_{\tau i} = 0$. We are then practically back to the baseline model after τ, independently of the rest of the model, has converged to $\tau^\star$.

In a complete description, the model in this section is made up of the following equations: (1) – (4) for the goods market equilibrium, (7) for desired utilization, (9) for the interest rate reaction function, (8) for the sentiment adjustments, (11) for the simple wage Phillips curve, (12) for the markup pricing, (22) for the adjustments of the markup rate, and (21) for the implied profit share.

Although this is quite a number of relationships, they can be reduced to two differential equations for the sentiment a and the markup rate τ. In a first step, we derive a price Phillips curve from the logarithmic differentiation of the price level in (12). Substituting the wage Phillips curve and the Taylor rule for the corresponding expressions, it becomes fairly different from eq. (10):

$$\pi \;\;=\;\; \pi^\star \;+\; \frac{(\beta_{\tau u}+\beta_{\tau i}\,\mu_u+\beta_w)(u-u^\star) \;\;-\;\; \beta_{\tau\tau}(\tau-\tau^\star)}{1 \;-\; \beta_{\tau i}\,(\mu_\pi-1)} \tag{23}$$

As further detailed in a remark on eq. (A9) in the appendix, in order for (23) to make economic sense it must be assumed that the denominator is positive. This puts a certain limit to the strength of the markup channel of monetary policy, $\beta_{\tau i}(\mu_\pi - 1) < 1$.

The new price Phillips curve (23) includes the gap between the two markup rates τ and $\tau^\star$ in addition to the familiar utilization gap and generally makes reference to the reaction coefficients in the markup adjustments. Furthermore, *via* the reactions of the markup to the interest costs, also the central bank's policy coefficients μ_u and μ_π come into play. Nevertheless, the main structural difference from the Phillips curve (10) in the baseline model is that *ceteris paribus* a higher markup reduces the inflation rate, which as a pure statement on its own could appear somewhat surprising at first sight. It might be conjectured that this feature will favour rather than impede stability. The Phillips curve is, however, not the only place where the markup takes effect. It is next seen that it also affects utilization and thus the changes in the sentiment variable.

By virtue of (23) and of course the Taylor rule, the real interest rate can be written as a function of $(u - u^\star)$ and $(\tau - \tau^\star)$. Substituting it in (7) for desired utilization and solving the market clearing condition $g^i = g^s$ for u yields IS utilization as the following function of the sentiment a and the markup τ,

$$\begin{aligned} u \;=\; u(a,\tau) \quad &= \quad \frac{a \,+\, \beta_d\,\gamma\,\widetilde{\mu}_{i\tau}\,(\tau-\tau^\star) \,-\, \gamma(1-\beta_d\,\widetilde{\mu}_{iu})\,u^\star}{sh(\tau) \,-\, \gamma \,+\, \beta_d\,\gamma\,\widetilde{\mu}_{iu}} \qquad (24) \\ \widetilde{\mu}_{iu} \quad &:= \quad [\,\mu_u \,+\, (\mu_\pi - 1)\,(\beta_{\tau u} + \beta_w)\,]\;/\;[\,1 \,-\, \beta_{\tau i}\,(\mu_\pi - 1)\,] \\ \widetilde{\mu}_{i\tau} \quad &:= \quad \beta_{\tau\tau}\,(\mu_\pi - 1)\;/\;[\,1 \,-\, \beta_{\tau i}\,(\mu_\pi - 1)\,] \end{aligned}$$

In the meantime it goes without saying that the denominator is positive and that an increase in the sentiment raises economic activity. The markup τ has a positive effect in the numerator and, *via* the profit share, a negative effect in the denominator. Normally, unless $\beta_d\gamma$ is excessively large, the latter effect will be dominant and so output may be said to be wage-led: a decrease of the markup (or profit share), which is tantamount to an increase in the wage share, leads to an increase in IS utilization. To limit the discussion, this is also what will be presupposed in the following.[31]

Turning to the dynamics, we express the real interest rate $(i - \pi)$, desired utilization u_d and the return differential $\rho = hu - (i - \pi)$ as functions of (u, τ). With $u = u(a, \tau)$, the economy can finally be reduced to two differential equations in a

[31]More specifically, one computes that $\partial u/\partial\tau < 0$ is equivalent to $\beta_d\gamma < su/[\widetilde{\mu}_{i\tau}(1+\tau)^2]$.

and τ:

$$\begin{array}{rcl} \dot{a} & = & \eta_u\,[\,u - u_d(u,\tau)\,] \;+\; \eta_\rho\,[\,\rho(u,\tau) - \rho^\star\,]\,, \qquad u = u(a,\tau) \\ \dot{\tau} & = & (1{+}\tau)\,[\,\theta_u(u-u^\star) \;-\; \theta_\tau(\tau-\tau^\star)\,] \\ \theta_u & := & \beta_{\tau u} \;+\; \beta_{\tau i}\,\widetilde{\mu}_{iu} \\ \theta_\tau & := & \beta_{\tau\tau} \;+\; \beta_{\tau i}\,\widetilde{\mu}_{i\tau} \end{array} \tag{25}$$

Regarding existence of a well-behaved steady state of (25), similar remarks as before can be made if the the normal rate of profit is set as $r^\star = h(\tau^\star)u^\star$; in this case the equalities $\dot{\tau} = 0$ and $u = u^o = u^\star$ imply $\tau^o = \tau^\star$, too. Assumption 1, however, has to be modified somewhat for the stability analysis. Its counterpart of the relative importance of the profitability motive *versus* the influence of the utilization gap, which so far was represented by the two coefficients η_ρ and η_u, now has to include the coefficients of the markup adjustments in addition. Reference will thus be made to

Assumption 2

The coefficient $\widetilde{\mu}_{iu}$ defined in (24) exceeds the profit share $h = h(\tau^o)$ in the long-run equilibrium, and the composite coefficient $\widetilde{\eta}_{\rho u} := \eta_\rho$ $(\widetilde{\mu}_{iu} - h) \;-\; \eta_u\,(1 - \beta_d\widetilde{\mu}_{iu})$ is positive. In addition, $\theta_u = \beta_{\tau u} \;+\; \beta_{\tau i}\,\widetilde{\mu}_{iu}$ ≥ 0.

Taking the discussion on the order of magnitude of $\mu_{u\pi}$ following Assumption 1 into account, the first inequality in Assumption 2 can be safely assumed to be satisfied if $b_{\tau u}$ in the definition of $\widetilde{\mu}_{iu}$ is positive or at least not too negative relative to μ_u, β_w and $\beta_{\tau i}$. The inequality $\theta_u \geq 0$ (which would be no problem if a strongly negative $b_{\tau u}$ is ruled out) is added in Assumption 2 in order to avoid too many case distinctions in the analysis.

Assumption 2 is important but not yet decisive for stability. The relationships between the structural coefficients ensuring stability are in fact rather involved. The formulation in Proposition 4, as before, focusses on the weight of the profitability *versus* the utilization gap motive in the sentiment dynamics. Interestingly, the direct autofeedback in the markup adjustments, represented by $\beta_{\tau\tau}$, plays a significant role, too.

Proposition 4

Suppose $\partial u/\partial\tau < 0$ for the reactions of IS utilization (24). In the special case when $\theta_u = \beta_{\tau u} + \beta_{\tau i}\,\widetilde{\mu}_{iu} = 0$, Assumption 2 is a necessary and *sufficient condition for stability. If $\theta_u > 0$, Assumption 2 is a necessary but not sufficient condition. Stability is then ensured if, and only if, in addition*

$$A = A(\eta_\rho) := \; \eta_\rho(\mu_{u\pi} - h) \; - \; \eta_u(1 - \beta_d \widetilde{\mu}_{iu}) \; > \; 0$$

and $\beta_{\tau\tau}$ is so large that

$$\beta_{\tau\tau} \; > \; B = B(\eta_\rho) := \; \eta_\rho \, \theta_u \, u^o \, [1 - \beta_{\tau i}(\mu_\pi - 1)] \, / \, [(1+\tau^o)^2 A(\eta_\rho)]$$

If instability prevails, it is of the saddle-point type.

In finer detail, the expression B is decreasing in η_ρ if $\eta_u > 0$, while B equals $\theta_u u^o/(\mu_{u\pi} - h)$ and is thus independent of η_ρ if $\eta_u = 0$).

It should first be emphasized that even in the complete absence of the utilization gap in the sentiment dynamics, when $\eta_u = 0$ and the original Harrodian destabilization mechanism has disappeared, the profitability motive is no longer strong enough to ensure stability. It must rather be supported by a sufficiently strong autofeedback in the markup. Regarding positive coefficients η_u it may be noticed that at least if $\beta_{\tau u} \geq 0$ (or mildly negative), the condition $A(\eta_\rho) > 0$ is more restrictive than $\widetilde{\eta}_{\rho u} > 0$ in Assumption 2. If the profitability motive is sufficiently pronounced to achieve $A > 0$ but, on the other hand, not overly strong, the term A will be relatively small and stability would require relatively large values of $\beta_{\tau\tau}$ in the second inequality of Proposition 4. These elementary observations show that there is considerable scope for instability. It might even be concluded that instability tends to be the rule rather than the exception.

Now, what is the reason for an unstable behaviour? It is instructive in this respect to have a look at the Jacobian matrix J of (25) and its sign pattern. First, mathematically, with Assumption 2 and $u_a = \partial u/\partial a > 0$, $u_\tau = \partial u/\partial \tau < 0$, $h' = dh/d\tau = 1/(1+\tau^o)^2 > 0$, we have

$$J \;=\; \begin{bmatrix} -\widetilde{\eta}_{\rho u} \, u_a & -\widetilde{\eta}_{\rho u} \, u_\tau + \; \eta_\rho(h' u^o + \widetilde{\mu}_{i\tau}) \\ (1+\tau) \, \theta_u \, u_a & (1+\tau) \, (\theta_u \, u_\tau - \theta_\tau) \end{bmatrix} \;=\; \begin{bmatrix} - & + \\ + & - \end{bmatrix} \tag{26}$$

Hence the autofeedbacks in the diagonal entries of a onto itself and τ onto itself are both stable. The determinant of J is positive, which is thus necessary and sufficient for stability, in the special case $\theta_u = 0$ (so that $j_{21} = 0$), or generally if the product of the two off-diagonal elements is smaller than the product of the entries on the diagonal; otherwise $\det J < 0$ and saddle-point instability obtains. The two inequalities in Proposition 4 are concerned with this point.

Second, for an economic reasoning, consider a positive shock to the markup τ in the steady state. This decreases utilization ($u_\tau < 0$), which in turn increases the return differential (since $\partial\rho/\partial u = h - \widetilde{\mu}_{iu} < 0$). Therefore, with a sufficiently strong profitability motive (i.e. $\widetilde{\eta}_{\rho u} > 0$ and consequently $-\widetilde{\eta}_{\rho u} u_\tau > 0$), the sentiment variable rises (this effect is captured by $j_{12} > 0$). The increase in a has subsequently a positive impact on utilization ($u_a > 0$) and on the whole also on the markup (formalized by $j_{21} > 0$). The little chain establishes a positive feedback loop of τ onto itself. Although compared to $j_{22} = \partial\dot{\tau}/\partial\tau < 0$ it is of a more indirect nature, it could nevertheless be dominant, namely, if $A < 0$ or $\beta_{\tau\tau} < B$ in the proposition. It may be mentioned that the instability thus arising is a conclusion that without a mathematical analysis, on the basis of a common sense argument alone, would have been hard to arrive at.

8 Conclusion

The paper was concerned with the effects of monetary policy in a Kaleckian model of growth and distribution facing Harrodian instability. The latter comes into play with a sentiment variable for the firms that, on the one hand, shifts the entire investment function and, on the other hand, reacts to the utilization gap. Regarding monetary policy, the literature was so far primarily interested in an incorporation of the basic idea of the New Consensus into such a setting. Accordingly, it assumes a direct negative impact of the real interest rate in the investment function. In contrast to this pedagogical and straightforward stability argument, a number of elements were proposed here that seek to take a better account of the policy transmission mechanisms in the real world. The two most important innovations are (i) a delayed effect of the interest rate on investment and thus economic activity, and (ii) reactions of the business sentiment not only to utilization but also profitability or, more precisely, to profitability compared to the risk-free real interest rate. Thus, instead of virtually suffocating the Harrodian mechanics by slipping the standard stability effects of the New Consensus over their head, so to speak, stability becomes an open issue again.

Three price inflation modules were considered in our framework. Common to them is a wage Phillips curve (though possibly with a zero slope) and the markup pricing of firms. In the first two versions the markup rate is constant and in the third one it dynamically reacts to, in particular, the real interest rate (in a positive way). In the wage Phillips curve of versions 1 and 3, the inflation climate (an expression that we prefer over expected inflation) is the central bank's constant target rate of inflation, while in version 2 it gradually adjusts towards a weighted average of target and current inflation.

Version 1, which is therefore our baseline model, is stable if and only if, in a certain sense, the profitability motive in the sentiment adjustments is stronger than the directly destabilizing Harrodian effect of the utilization gap. Perhaps somewhat surprisingly, the higher variability of inflation in the second model version, which is caused by a lower credibility of the central bank, widens the prospects for stability. By contrast, owing to a positive feedback loop in the cross-interactions between the sentiment and the markup rate, the third version narrows them.

The overall message of these results is that the central bank does indeed have some potential to stabilize the economy, even though the interest rate effects take some time to work out. On the other hand, monetary policy is not the stabilizing panacea it is usually thought to be in the New Consensus. The paper has demonstrated this for different assumptions about the inflation process, still within the confines of markup pricing and a wage Phillips curve. In this way we also get a feeling that for every new kind of feedback that may be incorporated into the present baseline model—for example, feedbacks arising from a debt financing of investment—the question has to be asked again whether this creates a channel for monetary policy that tends to exert an additional stabilizing or destabilizing influence.

Against this background we may conclude with the remark that an unstable long-run equilibrium could also be a theoretically welcome phenomenon—when we turn from the local to the global dynamics. With a nonlinear Phillips curve or a suitable nonlinearity in the sentiment dynamics, for example, the stabilizing interest rate feedback effects may be of different strength in a closer and wider neighborhood of the steady state, and this variability can provide a mechanism for the economy to exhibit persistent cyclical behaviour.[32] Monetary policy might then be said to be sufficiently stabilizing in the outer regions of the state space, a statement that could widen the perspective of the discussion of monetary policy.

Appendix

Derivation of IS utilization (13)

The Taylor rule (9) together with (10) can be used to express the real interest rate as a function of the two utilization gaps,

$$(i-\pi) \;-\; (i^\star - \pi^\star) \;\;=\;\; \mu_u\,(u - u^\star_{cb}) \;\;+\;\; (\mu_\pi - 1)\,\beta_w\,(u - u^\star_f) \qquad \text{(A1)}$$

Next, plug (A1) into the specification (7) of desired utilization u_d and, together with (1) for the profit rate $r = hu$, into the return differential $\rho = r - (i - \pi)$:

$$u_d \;=\; u_d(u)$$

[32]A fruitful and flexible proposal for the specification of a 'microfounded' sentiment dynamics is Franke(2012, 2014), which yields the aforementioned nonlinearity practically 'for free'.

$$
\begin{aligned}
&= u_f^\star \;+\; \beta_d\,[\,\mu_u\,(u-u_{cb}^\star) \;+\; (\mu_\pi-1)\,\beta_w\,(u-u_f^\star)\,] && \text{(A2)}\\
\rho \;&=\; \rho(u)\\
&= \rho^\star \;-\; (i^\star-\pi^\star+\rho^\star) \;+\; h\,u \;-\; \mu_u\,(u-u_{cb}^\star) \;-\; (\mu_\pi-1)\,\beta_w\,(u-u_f^\star) && \text{(A3)}
\end{aligned}
$$

Substituting (A2) in the market clearing condition (4), $g^i = g^s$, gives us an explicit solution for the IS utilization rate:

$$
u \;=\; u(a) \;=\; \frac{a \;+\; \beta_d\,\gamma\,\mu_u\,u_{cb}^\star \;+\; \gamma\,[\,\beta_d(\mu_\pi-1)\beta_w-1\,]\,u_f^\star}{sh \;-\; \gamma \;+\; \beta_d\,\gamma\,[\mu_u+(\mu_\pi-1)\beta_w]} \tag{A4}
$$

IS equilibrium when desired utilization is nonlinear

Let $u_d = u_d[(i-\pi)-(i^\star-\pi^\star)]$ be a general function with $u_d(0)=u_f^\star$ and derivative $\beta_d := u_d' \geq 0$ and consider the baseline model (the argument in the other model versions would be analogous). Use (A1) for the real interest rate gap and define $v = v(u) := u_d[\mu_u(u-u_{cb}^\star)+(\mu_\pi-1)\beta_w(u-u_f^\star)]$. Market clearing $g^s - g^i = 0$ can be written as

$$
f(a,u) \;:=\; shu \;-\; a \;-\; \gamma[u-v(u)] \;\;=\;\; 0
$$

This relationship is to read as an implicit equation for $u = u(a)$, so that $f[a,u(a)] \equiv 0$ for all a in a neighborhood of the steady state, where $a = a^o$. We have $f_a = \partial f/\partial a = -1$ and $f_u = \partial f/\partial u = sh - \gamma\{1-\beta_d[\mu_u+(\mu_\pi-1)\beta_w]\}$. Application of the Implicit Function Theorem then gives us

$$
du/da \;\;=\;\; -f_a\,/\,f_u \;\;=\;\; 1\,/\,\{sh \;-\; \gamma \;+\; \beta_d\gamma[\mu_u+(\mu_\pi-1)\beta_w]\}
$$

which is identical to the derivative of eq. (A4) with respect to a.

The central bank's reaction if the steady state does not support its target rates

As indicated in the text, suppose $u_{cb}^\star = u_f^\star$. Substituting (A2) and (A3) for u_d and ρ, using this in (14), setting the equation equal to zero and solving it for u yields a handsome expression for equilibrium utilization,

$$
u^o \;\;=\;\; u^\star \;+\; \frac{[\,r^\star - (i^\star-\pi^\star) - \rho^\star\,]\,\eta_\rho}{(\mu_{u\pi}-h)\,\eta_\rho \;-\; (1-\beta_d\,\mu_{u\pi})\,\eta_u}\;, \qquad r^\star := \;h\,u^\star \tag{A5}
$$

The denominator of the fraction is positive if Assumption 1 is satisfied. Now, consider a situation where the central bank underestimates the natural real rate of interest, $i^\star - \pi^\star < r^\star - \rho^\star$. This will cause persistent overutilization in the steady state, $u^o > u^\star$. Because, as it is readily seen, $u^o - u_d(u^o) = (1-\beta_d\,\mu_{u\pi})(u^o - u^\star)$, the firms' utilization gap is positive, too. On the other hand, owing to (A1) the overutilization will be associated with a real interest rate $(i^o - \pi^o)$ in the steady state that exceeds the target $(i^\star - \pi^\star)$.

It was suggested that the latter observation leads the central bank to conclude that it has missed the 'natural' real interest rate and set too low a target. Correspondingly, it adjusts $(i^\star - \pi^\star)$ upwards. This decreases the fraction in (A5) and so reduces the gap between u^o and $u^\star$. By the same token, the difference between $(i^o - \pi^o)$ and $(i^\star - \pi^\star)$ in (A1) decreases. The two rates would eventually align if this procedure is gradually continued, that is, the central bank's adjustments do operate in the correct direction. Note, however, that the reactions would work in the opposite direction and the gaps would widen if Assumption is violated and therefore the denominator in (A5) is negative.

Proof of Proposition 3

Using (17) in the Taylor rule gives the following expression for the real interest rate,

$$\begin{aligned} i-\pi &= (i-\pi)(u,\pi^c) \;=\; i^\star - \pi^\star \\ &\quad + \; (\mu_\pi - 1)\,(\pi^c - \pi^\star) \;+\; \mu_u\,(u - u^\star_{cb}) \;+\; (\mu_\pi - 1)\,\beta_w\,(u - u^\star_f) \end{aligned} \tag{A6}$$

Substituting this in desired utilization (7) and solving the market clearing condition $g^i = g^s$ for u yields

$$\begin{aligned} u &= u(a,\pi^c) \\ &= \frac{a \;-\; \beta_d\gamma(\mu_\pi - 1)(\pi^c - \pi^\star) \;+\; \gamma\{\beta_d\mu_u u^\star_{cb} + [\beta_d(\mu_\pi - 1)\beta_\pi - 1]u^\star_f\}}{sh \;-\; \gamma \;+\; \beta_d\gamma[\mu_u + (\mu_\pi - 1)\beta_w]} \end{aligned} \tag{A7}$$

By lastly substituting the real interest rate in the return differential, system (20) is obtained. With $u_a := \partial u/\partial a = 1/\sigma > 0$, $u_\pi = \partial u/\partial \pi^c = -\beta_d\gamma(\mu_\pi - 1)/\sigma < 0$ (σ being the denominator in (A7)) and $\eta_{\rho u}$ as defined in the Proposition 3, the Jacobian matrix is calculated as

$$J \;=\; \begin{bmatrix} -\eta_{\rho u}\, u_a & -\eta_{\rho u}\, u_\pi \;-\; (\mu_\pi - 1)(\eta_\rho + \eta_u\beta_d) \\ \alpha_\pi(1-\chi)\beta_w\, u_a & \alpha_\pi[-\chi \;+\; (1-\chi)\beta_w\, u_\pi] \end{bmatrix} \tag{A8}$$

The determinant results like $\det J/\alpha_\pi u_a = \eta_{\rho u}\chi \;+\; (\mu_\pi - 1)\,[(1-\chi)\beta_w(\eta_\rho + \eta_u\beta_d)]$ (the partial derivative u_π cancels out), while the trace is obvious. The first inequality in the proposition is equivalent to a negative trace (use the formula for u_π and solve $\operatorname{trace} J < 0$ for $\eta_{\rho u}$), and the second to $\det J > 0$.

Analysis of the dynamic markup adjustments

Substitute the Taylor rule written as $(i-\pi) - (i^\star - \pi^\star) = (\mu_\pi - 1)(\pi - \pi^\star) + \mu_u(u-u^\star)$ in (22) and use this equation as well as the wage Phillips curve (11) for the logarithmic differentiation of the price level in (12). In this way we get

$$\begin{aligned} \pi \;&=\; \hat{p} \;=\; \dot{\tau}/(1+\tau) \;+\; \hat{w} \;-\; \hat{z} \\ &=\; \beta_{\tau u}(u-u^\star) \;+\; \beta_{\tau i}\,[(\mu_\pi - 1)(\pi-\pi^\star) + \mu_u(u-u^\star)] \\ &\quad -\; \beta_{\tau\tau}(\tau-\tau^\star) \;+\; \pi^\star \;+\; \beta_w(u-u^\star) \end{aligned} \tag{A9}$$

Solving this equation for the inflation gap $(\pi - \pi^\star)$ yields (23) in the text. To be scrupulous, the inflation rate on the left-hand side of (A9) is a forward derivative of the price level and the one on the right-hand side a backward derivative. Assuming that they coincide should be compatible with the idea that higher values of $\beta_{\tau i}(\mu_\pi - 1)$ in (A9) accelerate the inflation process, while not reversing the reactions of π to the utilization and markup gaps. Hence the denominator in (23) must be positive, $\beta_{\tau i}(\mu_\pi - 1) < 1$.[33]

With $(\pi - \pi^\star)$ from (23) in the expression for the real interest rate gap and a little calculation we have

$$(i-\pi) \;=\; (i-\pi)(u,\tau) \;=\; (i^\star - \pi^\star) \;+\; \widetilde{\mu}_{iu}(u-u^\star) \;-\; \widetilde{\mu}_{i\tau}(\tau-\tau^\star) \tag{A10}$$

where $\widetilde{\mu}_{iu}$ and $\widetilde{\mu}_{i\tau}$ are defined as in (24). The derivation of (24) itself from the IS condition $shu = g^s = g^i = a + \gamma\{u - u^\star - \beta_d\,[(i-\pi) - (i^\star - \pi^\star)]\}$ is then straightforward.

Proof of Proposition 4

From the entries of the Jacobian matrix (26) one computes (omitting the superscript 'o' for the steady state values)

$$\begin{aligned} H \;&:=\; \det J/(1+\tau)\,u_\tau \\ &=\; -\widetilde{\eta}_{\rho u}\,\theta_u\,u_\tau \;+\; \widetilde{\eta}_{\rho u}\,\theta_\tau \;+\; \widetilde{\eta}_{\rho u}\,\theta_u\,u_\tau \;-\; \eta_\rho\,\theta_u\,(h'u + \widetilde{\mu}_{i\tau}) \\ &=\; \widetilde{\eta}_{\rho u}\Big[\beta_{\tau\tau} \;+\; \frac{\beta_{\tau i}\,\beta_{\tau\tau}\,(\mu_\pi - 1)}{1-\beta_{\tau i}(\mu_\pi - 1)}\Big] \;-\; \eta_\rho\,\theta_u\,h'u \;-\; \eta_\rho\,\theta_u\,\frac{\beta_{\tau\tau}\,(\mu_\pi - 1)}{1-\beta_{\tau i}(\mu_\pi - 1)} \end{aligned}$$

[33] Admitting the denominator in (23) to become negative would not only appear economically doubtful but may also cause misleading stability or instability conclusions; for a similar phenomenon, see the alternative model versions of a Harrodian dynamics discussed in Section 4 of Franke (2015).

$$= \quad \frac{\beta_{\tau\tau}}{1-\beta_{\tau i}(\mu_\pi - 1)}\,[\,\widetilde{\eta}_{\rho u} - \eta_\rho\,\theta_u\,(\mu_\pi - 1)\,] \;-\; \eta_\rho\,\theta_u\,u/(1+\tau)^2$$

Using the definitions of $\widetilde{\eta}_{\rho u}$, θ_u, $\widetilde{\mu}_{iu}$ and finally $\mu_{u\pi}$, the term in square brackets is seen to be equal to $A = \eta_\rho(\mu_{u\pi} - h) - \eta_u(1 - \beta_d\,\widetilde{\mu}_{iu})$. Clearly, for H and thus det J to be positive, A must be positive and, multiplied by the fraction in front of it, larger than $\eta_\rho\,\theta_u\,u/(1+\tau)^2$. The condition $\beta_{\tau\tau} < B$ solves this inequality for $\beta_{\tau\tau}$.

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