Does Classical Competition Explain the Statistical Features of Firm Growth?

Simone Alfarano and Mishael Milaković

Dept. of Economics, University of Kiel, Olshausenstr. 40, 24118 Kiel, Germany

Abstract

We express the idea of classical competition in a statistical equilibrium model, where the tendency for competition to equalize profit rates results in an exponential power (or Subbotin) distribution. The model supports and extends recent evidence on the Laplace distribution of growth rates in firm size. We also find tent-shaped distributions in the size growth rates of *Forbes Global 2000* companies, which we interpret as preliminary evidence in favor of the hypothesis that classical competition is a globally operating mechanism.

Key words: Statistical equilibrium, classical competition, maximum entropy, profit rates, firm growth rates, Subbotin distribution, Laplace distribution
JEL codes: C16, L10, D21, E10, F01

1 Introduction

The classical notion of competition (see Smith, 1776, Chapter IX) rests on the idea that entrepreneurs will allocate their capital into the most profitable sector or business activity, utilizing the average rate of profit as a benchmark in their investment decision. We propose a statistical equilibrium model in the spirit of Foley (1994) that interprets the tendency for competition to equalize profit rates as a dispersion measure around an average profit rate. The model predicts an *exponential power* or *Subbotin* distribution of firms' profit rates. Obviously, profit rates and firm growth rates should be tightly linked, and in a first approximation we abstract from real frictions and time-lag structures, taking firm growth rates as synonymous with profit rates. Consequently, if profit rates are reflected in the financial structure of firms, the prediction of a Subbotin distribution supports and extends recent evidence on the Laplace distribution of growth rates in firm size (see, for instance, Bottazzi and Secchi, 2003, 2006; Bottazzi et al., 2001; Stanley et al., 1996). Hence our model links the empirical regularities in the distribution of firm growth rates to the idea of classical competition.

2 Statistical Equilibrium Model of Classical Competition

Classical competition essentially describes a negative feedback mechanism. Capital will seek out sectors or industries where the profit rate is higher than the economy-wide average, typically attracting labor, raising output, and reducing prices and profit rates, which in turn provides an incentive for capital to leave the sector, thereby leading to higher prices and profit rates for firms that remain in the sector (see Foley, 2006). As a result classical competition tends to equalize profit rates, yet continually changing tastes, technologies, and entry and exit dynamics render a complete elimination of differences in sectoral profit rates improbable. We take the position that the average profit rate corresponds to a measure of central tendency, while the complex movements of capital in search of profit rate equalization translate into a measure of dispersion around the average. Algebraically, a quite general way to formulate such a dispersion measure is the *standardized α-th moment around m*, $\sigma^\alpha = E|x-m|^\alpha$, where x denotes profit or growth rates, and $m \in \mathbb{R}$ is the average profit rate. In order to determine the statistical equilibrium outcome of such a view of classical competition, we employ the *maximum entropy principle*. From the viewpoint of probability theory, the maximum entropy principle produces the informationally least biased distribution of a random variate if our knowledge is encoded in moment constraints. Alternatively, the principle yields the combinatorially mostly likely distribution, because it maximizes the multiplicity of feasible assignments given the moment constraints (see Jaynes, 1978, for a comprehensive account of the maximum entropy principle).

Formally, our view of classical competition leads to a *statistical equilibrium distribution* (see, e.g., Castaldi and Milaković, 2007; Foley, 1994) if we consider the continuous maximum entropy program with objective function

$$\max_{f(x)} H \equiv -\int_{-\infty}^{+\infty} f(x) \log f(x)\ dx, \tag{1}$$

subject to the natural constraint that normalizes the growth rate density $f(x)$,

$$\int_{-\infty}^{+\infty} f(x)\ dx = 1\,, \tag{2}$$

and subject to the moment constraint on the dispersion of growth rates,

$$\int_{-\infty}^{+\infty} f(x) \left|\frac{x-m}{\sigma}\right|^\alpha\ dx = 1\,. \tag{3}$$

From an economic point of view, the outcome of the particular maximum entropy program (1)–(3) yields the growth rate distribution arising from the *most decentralized activity of competitive firms* that utilize m as a benchmark profit rate in their investment and operating decisions. We show in the appendix that the solution to the variational problem (1)–(3) is given by the Subbotin distribution,

$$f(x; m, \sigma; \alpha) = \frac{1}{2\sigma\alpha^{1/\alpha}\Gamma(1 + 1/\alpha)} \exp\left(-\frac{1}{\alpha}\left|\frac{x-m}{\sigma}\right|^{\alpha}\right), \qquad (4)$$

where $\Gamma(\cdot)$ is the Gamma function. The (symmetric) Subbotin distribution is characterized by a location parameter m, a scale parameter $\sigma > 0$, and a shape parameter $\alpha > 0$. If α is smaller (greater) than two, the distribution is leptokurtic (platykurtic). Notice that for $\alpha = 1$, the Subbotin distribution reduces to the Laplace distribution, for $\alpha = 2$ it reduces to the Gaussian, for $\alpha \to \infty$ it tends to a uniform, and for $\alpha \to 0$ the distribution degenerates to a Delta, so the microeconomic textbook concept of a competitive equilibrium characterized by some fixed point m, is also included as a special case of the statistical equilibrium model. [1]

3 Application to the Forbes Global 2000

Previous empirical evidence on firm growth rate distributions includes findings on all publicly traded US manufacturing companies in the COMPUSTAT database (see Stanley et al., 1996), on the PHID database of the world's largest pharmaceutical companies (see Bottazzi et al., 2001), and on the MICRO.1 database containing sectorally disaggregated Italian manufacturing firms (see Bottazzi and Secchi, 2003, 2006). These studies find that α is usually close to unity, indicating a Laplace distribution, regardless of whether one considers sectorally aggregated or disaggregated data within a country, or worldwide data within a sector.

The *Forbes Global 2000* list of the world's largest companies provides an expeditious opportunity to check whether the results carry over across sectors as well as across countries. [2] Let $S_i(t)$ represent sales or market values of the ith company in the list at time t, and denote the log growth rate by $x_i(t) = \log S_i(t) - \log S_i(t-1)$; then the annual *normalized* growth rate is $g_i(t) = x_i(t) - \langle x_i(t) \rangle$, where $\langle x_i(t) \rangle = (\sum_{i=1}^{N} \log S_i(t))/N - (\sum_{i=1}^{N} \log S_i(t-1))/N$. We have $N = 1712$ growth rate observations for 2005-06, and $N = 1658$ observations for 2006-07, because the composition of the Forbes list changes between

[1] Actually, this special case is the most improbable of all possible results because it has a multiplicity of unity.

[2] The list is publicly available at `www.forbes.com/global2000`.

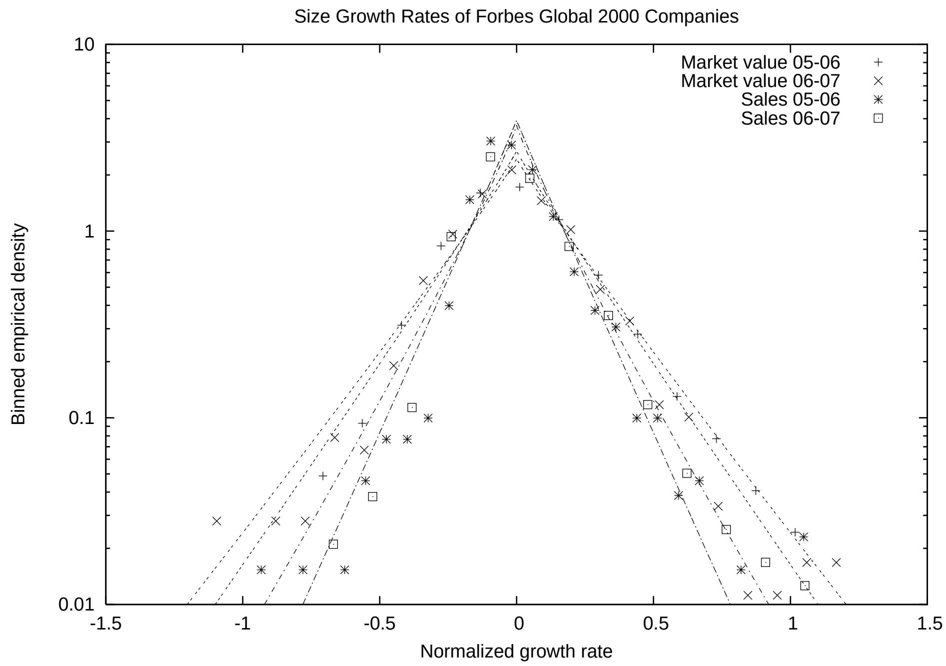

Fig. 1. Empirical densities of normalized size growth rates for the *Forbes Global 2000* list of the world's largest companies.

	m	σ	α	L_U/N	L_R/N	p
Sales 05-06	0.111	0.129	0.98	0.34755	0.34748	0.640
	(0.005)	(0.004)	(0.04)			
Sales 06-07	0.115	0.144	0.90	0.20017	0.19758	0.003
	(0.006)	(0.005)	(0.04)			
MV 05-06	0.156	0.212	0.95	−0.16065	−0.16121	0.166
	(0.008)	(0.007)	(0.04)			
MV 06-07	0.142	0.191	0.96	−0.05552	−0.05587	0.283
	(0.007)	(0.006)	(0.04)			

Table 1
Maximum likelihood estimates of the Subbotin parameters (standard errors in parentheses). The log-likelihoods per point for the restricted (Laplace) and unrestricted (Subbotin) model are denoted by L_R/N and L_U/N. The last column shows the p-value of the likelihood ratio test statistic with one degree of freedom.

years. Figure 1 shows the binned empirical densities of $g_i(t)$, which display the characteristic tent-shape of a Laplace distribution on semi-log scale. Maximum likelihood estimates of the Subbotin parameters obtained from $x_i(t)$ using the Subbotools package of Bottazzi (2004) are reported in Table 1. In three out of the four cases, a likelihood ratio test identifies the Laplace as the most parsimonious description of the data among the family of Subbotin distributions. Therefore, the evidence speaks in favor of a Laplace distribution on a global cross-sectoral level. In the context of our statistical equilibrium model, these findings would be consistent with a tendency for competition to globally equalize profit rates across different sectors.

4 Discussion and Conclusion

So far the Subbotin distribution has been used as an ad hoc estimation device without theoretical underpinning in the industrial dynamics literature (see, e.g., Bottazzi and Secchi, 2006). Thus the central contribution of our statistical equilibrium model is a theoretical justification for the Subbotin distribution based on the idea of classical competition. Structural differences in the statistical equilibrium model stem from differences in the shape parameter α, because operating on the location or scale parameters does not change the qualitative features of the equilibrium distribution. Hence, *ceteris paribus*, structural differences in the competitive environment created by interacting firms should ultimately be reflected in the shape parameter.

It is quite remarkable that the empirically observed shape parameters mostly speak in favor of a Laplace distribution, and several mechanisms have already been put forward to explain the Laplace distribution of firm growth rates, for instance the agent-based financial fragility model of Delli Gatti et al. (2005), or the increasing returns model of Bottazzi and Secchi (2006). In principle, however, there should be a plethora of different mechanisms that lead to a Laplace distribution. From a purely formal point of view, the maximum entropy principle informs us that irrespective of the peculiar features of a mechanism, the macroscopic dispersion constraint responsible for a Laplace distribution ($\alpha = 1$) will be a *linear measure of absolute deviations*, for instance the mean or median absolute deviation. From the viewpoint of our statistical equilibrium model, the central question should be What does a value of α close to unity imply about the competitive environment that firms and investors are facing? Therefore, a distinct feature of the model is that it provides a natural framework for interpreting rare yet statistically significant deviations from $\alpha = 1$ (see, e.g., Bottazzi and Secchi, 2006; Pammolli et al., 2007) as reflections of qualitative changes in the competitive environment.

References

Bottazzi, Giulio. 2004. *Subbotools User's Manual.* Laboratory of Economics and Management, LEM Papers Series, 2004/14, Sant'Anna School of Advanced Studies, Pisa, Italy.

Bottazzi, Giulio, Giovanni Dosi, Marco Lippi, Fabio Pammolli, and Massimo Riccaboni. 2001. Innovation and corporate growth in the evolution of the drug industry. *International Journal of Industrial Organization* 19:1161–1187.

Bottazzi, Giulio, and Angelo Secchi. 2003. Why are distributions of firm growth rates tent-shaped? *Economics Letters* 80:415–420.

———. 2006. Explaining the distribution of firm growth rates. *RAND Journal of Economics* 37(2):235–256.

Castaldi, Carolina, and Mishael Milaković. 2007. Turnover activity in wealth portfolios. *Journal of Economic Behavior & Organization* 63(3):537–552.

Delli Gatti, Domenico, Corrado Di Guilmi, Edoardo Gaffeo, Gianfranco Giulioni, Mauro Gallegatti, and Antonio Palestrini. 2005. A new approach to business fluctuations: Heterogeneous interacting agents, scaling laws and financial fragility. *Journal of Economic Behavior & Organization* 56:489–512.

Foley, Duncan K. 1994. A statistical equilibrium theory of markets. *Journal of Economic Theory* 62(2):321–345.

———. 2006. *Adam's Fallacy*. Cambridge, MA: Harvard University Press.

Jaynes, Edwin T. 1978. Where do we stand on maximum entropy? In *E. T. Jaynes: Papers on Probability, Statistics and Statistical Physics*, ed. Roger D. Rosenkrantz. Dordrecht, Netherlands: Kluwer Academic Publishers.

Pammolli, Fabio, Dongfeng Fu, Sergey V. Buldyrev, Massimo Riccaboni, Kaushik Matia, Kazuko Yamasaki, and H. Eugene Stanley. 2007. A generalized preferential attachment model for business firms growth rates. *European Physical Journal B* 57:127–130.

Smith, Adam. 1776. *An Inquiry into the Nature and Causes of The Wealth of Nations*. Petersfield, England: Harriman House (edition published in 2007).

Stanley, Michael H.R., Luis A.N. Amaral, Sergey V. Buldyrev, Heiko Leschhorn, Philipp Maass, Michael A. Salinger, and H. Eugene Stanley. 1996. Scaling behavior in the growth of companies. *Nature* 379:804–806.

A Maximum Entropy Derivation of the Subbotin Distribution

The Lagrangian of the maximum entropy program (1)–(3) is

$$\begin{aligned}\mathcal{L} = &-\int_{-\infty}^{\infty} f(x)\log f(x)\ dx - \mu\left[\int_{-\infty}^{\infty} f(x)\ dx - 1\right]\\ &-\lambda\left[\int_{-\infty}^{\infty}\left|\frac{x-m}{\sigma}\right|^{\alpha} f(x)\ dx - 1\right]\ ,\end{aligned}$$

where μ and λ denote the multipliers. The first order condition

$$\frac{\partial \mathcal{L}}{\partial f(x)} = -\log f(x) - \xi - \lambda \left| \frac{x-m}{\sigma} \right|^{\alpha} = 0 \, , \tag{A.1}$$

where $\xi \equiv 1 + \mu$, implies that the solution will have the functional form

$$f(x) = \exp(-\xi) \cdot \exp\left(-\lambda \left| \frac{x-m}{\sigma} \right|^{\alpha} \right) \, . \tag{A.2}$$

Therefore, in order to compute the values of the multipliers λ and ξ as a function of the parameters m, σ and α, we have to invert the constraints

$$\int_{-\infty}^{\infty} \exp(-\xi) \cdot \exp\left(-\lambda \left| \frac{x-m}{\sigma} \right|^{\alpha} \right) dx = 1 \tag{A.3}$$

and

$$\int_{-\infty}^{\infty} \exp(-\xi) \cdot \exp\left(-\lambda \left| \frac{x-m}{\sigma} \right|^{\alpha} \right) \left| \frac{x-m}{\sigma} \right|^{\alpha} \, dx = 1 \, . \tag{A.4}$$

Integrating by substitution, and using the definition of the Gamma function, we end up with

$$\exp(-\xi) = \frac{1}{2\sigma} \frac{1}{\alpha^{1/\alpha} \Gamma(1 + 1/\alpha)} \, , \tag{A.5}$$

and

$$\lambda = \frac{1}{\alpha} \, . \tag{A.6}$$

Notice that (A.5) is the *partition function* that normalizes the density. Finally, substituting for (A.5) and (A.6) in (A.2) leaves us with (4) in the main text.