A Minimal Noise Trader Model with Realistic Time Series Properties

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Abstract

Simulations of agent-based models have shown that the stylized facts (unit-root, fat tails and volatility clustering) of financial markets have a possible explanation in the interactions among agents. However, the complexity, originating from the presence of non-linearity and interactions, often limits the analytical approach to the dynamics of these models. In this paper we show that even a very simple model of a financial market with heterogeneous interacting agents is capable of reproducing realistic statistical properties of returns, in close quantitative accordance with the empirical analysis. The simplicity of the system also permits some analytical insights using concepts from statistical mechanics and physics. In our model, the traders are divided into two groups: *fundamentalists* and *chartists*, and their interactions are based on a variant of the herding mechanism introduced by Kirman [22]. The statistical analysis of our simulated data shows long-term dependence in the auto-correlations of squared and absolute returns and hyperbolic decay in the tail of the distribution of the raw returns, both with estimated decay parameters in the same range like empirical data. Theoretical analysis, however, excludes the possibility of 'true' scaling behavior because of the Markovian nature of the underlying process and the finite set of possible realized returns. The model, therefore, only *mimics* power law behavior. Similarly as with the phenomenological volatility models analyzed in LeBaron [25], the usual statistical tests are not able to distinguish between true or pseudo-scaling laws in the dynamics of our artificial market.

Keywords: Herd Behavior; Speculative Dynamics; Fat Tails; Volatility Clustering.
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1 Introduction

In the last couple of years, the study of behavioral models of dynamic interaction in financial markets has brought about a better understanding of some of the key stylized features of financial data, namely the *fat tails* of the distribution of returns and the *autoregressive dependence* in volatility. Although these statistical features have counted as almost universal findings for practically all financial time series for a long time and appear to be extremely uniform across assets and sampling horizons, economic explanations of their behavioral origins were nonexistent until very recently. However, the recent rush of interest in heterogeneous agents models with a diversity of interacting traders, the availability of fast computers for simulations of markets with a large number of agents, and the introduction of new analytical and computational tools (often adapted from statistical physics) in the analysis of multi-agent systems has led to a variety of new contributions in which the above stylized facts (one of them or both) are shown to be emergent properties of interacting agent dynamics. Some of these contributions show that besides the realistic distributional properties of their models, the overall dynamics is also undistinguishable from a unit-root process. Hence, despite having identifiable behavioral roots (in terms of the assumed speculative behavior of the agents), no immediately recognizable traces of predictability can be found in the presented time series, and the dynamics appears to be observationally equivalent to a martingale process.

Early papers in this area have often been the results of collaborations between economists and physicists, e.g. Takayasu *et al.* [34], Palmer *et al.* [32] and Bak *et al.* [4]. While they made important contributions to get this literature started, the similarity of the resulting time paths with empirical data was limited. Later studies have merged this multi-agent approach with the type of noise traders - fundamentalists' interaction introduced by Beja and Goldman [5] and Day and Huang [10]. Papers along this line included the microscopic stock market models of Lux and Marchesi [29, 30], Chen *et al.* [8, 9], Iori [21], Farmer [13], LeBaron [24] as well as the adapting belief dynamics of Gaumersdorfer and Hommes [15, 16]. A related variant can be found in the artificial foreign exchange markets of Arifovic *et al.* [3] and Georges [17] in which agents' behavior is modelled using genetic algorithmic (GA) for their selection of strategies.

Interestingly, some possible general explanations seem to emerge from this literature: first, volatility clustering and fat tails may emerge from indeterminacy in the equilibrium of the dynamics. In particular, with different strategies performing equally well in some kind of steady state, stochastic disturbances lead to continuously changing strategy configurations which at some point generate bursts of activity. This type of dynamics can be found already in Youssefmir and Huberman [36] in the context of a resource exploitation model and can be identified in both the papers by Lux and Marchesi [29, 30] and the otherwise quite different GA models by Arifovic and Gencay [3], Lux and Schornstein [31], and Georges [17].

Another more general approach can be attributed to Gaumersdorfer and Hommes [15], who show that volatility clustering can also emerge from stochastic dynamics with multiple attractors. Small amounts of noise added to a deterministic dynamics with two or more attractive states can lead to recurrent switches between these attractors. As these different regimes often have different degrees of volatility, volatility clustering is a somehow natural result of these dynamics. Interestingly, both of these mechanisms are sometimes identified as examples of **intermittency** which might, therefore, be thought of as a general conceptual framework for the explanation of the particular characteristics of financial markets.

While the above results have - due to their origin from the behavioral finance literature - more or less complicated descriptions of agents' expectations and strategy choice, physicists have rather tried to reduce the dynamics to a few basic principles able to generate the required time series characteristics. Recent models with only a few ingredients for activation and frustration of agents leading to realistic simulated output include Eguiluz and Zimmermann [12], Bornholdt [6] and variants of the so called minority game (Challet *et al.* [7]). Our aim in this paper is similar to these studies: we are interested in whether an extremely simplified model of interaction of noise traders and fundamentalists is already sufficient to reproduce the key stylized facts: unit roots, fat tails and volatility clustering. The model we investigate in this paper is a simple variant of the herding dynamics introduced by Kirman [22] and Lux [27]. We distinguish between two groups and allow for mimetic contagion among agents by simply postulating that they will move from one group to the other, with a certain probability depending on group size. This leads to the natural emergence of majority opinion with all agents sharing one of two available opinions. However, the stochasticity of the dynamics also leads to recurrent switches between both opinions, so that we find a bistable system with a bimodal ergodic distribution of states. Adding a simple price adjustment rule, bi-modality carries over to prices as well. Simulations of this model show that it can *mimic* in surprising quantitative accuracy the above stylized facts. The simplicity of the model also allows some analytically insights into its dynamics. In particular, it is straightforward to show that the model does not exhibit 'true scaling', neither concerning the distribution of large returns, nor the temporal dependence structure. The apparent scaling, in fact, results from a kind of 'regime switching' between the two modes of its ergodic stationary distribution. This is a phenomenon similar to the difficulty of distinguishing between apparent and true scaling in certain stochastic processes [1, 19, 11, 25]. Our analysis thus demonstrates that 'apparent' scaling is not confined to a particular class of appropriately constructed stochastic models, but might also prevail in behavioral models of interacting agents.

2 A Simple Model of Contagion

Our market is populated by N agents, each of them being either in state A or in state B. The number of agents in both groups are denoted N_A and N_B. The state of the system can be described by an intensive variable:

$$x = \frac{N_A - N_B}{N} \tag{1}$$

Every agent has a probability of switching per unit of time from one state to the other regulated by the transition rates:

$$\phi_{A\to B} = \nu \frac{N_B}{N} \qquad \phi_{B\to A} = \nu \frac{N_A}{N}. \tag{2}$$

where ν is a time-scaling parameter and

$$N_A = \frac{1+x}{2} N \qquad N_B = \frac{1-x}{2} N. \tag{3}$$

The number of agents that change state per unit of time is:

$$\omega_{A\to B} = \nu \frac{N_B}{N} N_A \qquad \omega_{B\to A} = \nu \frac{N_A}{N} N_B. \tag{4}$$

The previous transition rates with the finite realizations of x specify a reversible Markov chain, therefore the detailed balance condition holds [2]:

$$\omega(x \to x + \Delta x) P_e(x) = \omega(x + \Delta x \to x) P_e(x + \Delta x) \tag{5}$$

where the subscript e denotes the stationary probability distribution, that can be written as a Gibbs distribution [2]:

$$P_e(x) \propto exp^{U(x)} \tag{6}$$

Using (5) and (6), we have:

$$exp^{[U(x+\Delta x) - U(x)]} = \frac{(1-x)(1+x)}{[1-(x+\Delta x)][1+(x+\Delta x)]} \tag{7}$$

where

$$\Delta x = \frac{2}{N} \tag{8}$$

For large N [1], we can rewrite (7) in the limit:

$$\Delta x \to 0 \tag{9}$$

At the end, the result is a simple differential equation for $U(x)$:

$$\frac{dU(x)}{dx} = -\frac{d}{dx} \ln[(1-x)(1+x)] \qquad U(x) = -\ln(1-x^2) + c \tag{10}$$

From (6), it is straightforward to derive the equilibrium distribution:

$$P_e(x) = \frac{1}{L} \frac{1}{1-x^2} \tag{11}$$

where the normalization constant L is given by:

$$L = \int_{-1+\varepsilon}^{1-\varepsilon} \frac{1}{1-x^2} dx = \ln \frac{2-\varepsilon}{\varepsilon} \tag{12}$$

[1] In our simulations $N = 100$

ε is a positive number [2].

The system is bistable and generates transitions between the two 'equilibria' $x \approx +1$ and $x \approx -1$; we can derive the first passage time T_0 [2]:

$$T_0 = \sum_{x=-1+\varepsilon}^{1-2\varepsilon} \frac{1}{P_e(x) w(x \to x + \Delta x)} \sum_{y=x+\Delta x}^{1-\varepsilon} P_e(y). \tag{13}$$

In the limit of a large number of agents, the previous sum can be written as an integral; therefore we can calculate explicitly the first passage time:

$$T_0 = N \ln(N) + o(\varepsilon). \tag{14}$$

This model can be used to simulate interaction between economic agents (traders) based on **imitative behavior**.

The previous mechanism is inspired by Kirman's analysis of opinion formation (Kirman, [22]). The main difference with respect to Kirman's model is the absence of a constant term in the probability transitions (2), introduced by the author to prevent the absorbing states at $|x| = 1$; we replaced it by setting reflecting boundary conditions. Consequently, the only possible scenario is a distribution concentrated in the extreme values (U shape distribution), while in the later case a flat distribution and a distribution with a pick around the mean are also possible, depending on the particular set-up of the parameters.

3 The Financial Market Model

We now use this two-state dynamics as the main ingredient in a financial market model with interacting heterogenous agents. Our market participants are divided into two groups:

N_F *fundamentalists* (**F**), who buy (sell) a fixed amount of stocks T_F when the price is below (above) its fundamental value p_F.

N_C *noise traders* (**C**), who are driven by herd instinct.

Depending on their expectation about future price movements, noise traders can be either *optimists* (buyers or O) or *pessimists* (sellers or P). T_C represents the fixed number of stocks that noise traders buy or sell. While the number of F and C is constant in time (i.e. there are no switches between them), switches from O to P and vice versa are allowed. The two-state model, detailed in sec.2, regulates the transition rates. The fundamental price is assumed to be constant in time.

Assuming sluggish price adjustment, the dynamics of the price is given by

$$\frac{dp}{pdt} = \beta[N_F T_F (p_F - p) + N_C T_C x] \qquad x = \frac{N_O - N_P}{N_C} \tag{15}$$

[2] In our simulation, we leave at least one agent in each group to avoid total extinction, i.e. we implemented reflecting boundaries at $x = 1$ and $x = -1$; so $\varepsilon = \frac{2}{N}$.

where β is the reaction parameter of the market (speed of price adjustment).

As an approximation to the resulting non-equilibrium dynamics we assume instantaneous market clearing (which can be interpreted as an adiabatic approximation in physics terminology). We can solve (15) for the equilibrium price:

$$p = p_F + \frac{N_C T_C}{N_F T_F} x. \tag{16}$$

Without loss of generality, we focus attention on the following set of parameters values:

$$N_C = N_F = N \qquad T_C = T_F = 1 \qquad \beta = 1. \tag{17}$$

By (16), the average of the price is p_F because the mean of x is zero. We can observe, however, pessimistic phases in which the asset is undervalued (compared to the fundamental price), and switching to optimistic phases in which it is overvalued. In the first case the majority of the noise traders is in the 'pessimistic mood', while the second case most of them are in an 'optimistic mood'.

This model can be viewed as a simplified version of the more complex artificial stock market introduced by Lux and Marchesi [29]. However, their switching mechanism is influenced by two factors: the opinion of others, modelled via an opinion index similar to our x, and the local dynamics of the price, entering via the averaged trend. In addition to this, the complexity is increased by the possibility of switching among three different states and not only two as in our model.

[Insert figures 1 and 2 approximately here]

As it turns out, the model is able to reproduce some of the salient characteristics of financial markets. Figures 1 and 2 illustrate the results of the model. Volatility clusters are visible in the time series of returns in correspondence to the change of majority, and the unconditional distribution of returns seems to be leptokurtic. Dependence of absolute and squared returns (as a measure of the volatility) is positive over an extended time horizon, while the raw returns show almost no correlation. All these features are in qualitative agreement with empirical findings.

4 Statistical Analysis of Simulated Data

In order to see how closely the statistical results from our simulated data match empirical observations, we performed a series of experiments with a long data set of $1,000,000$ integer time steps. Tables 1 and 2 give some elementary statistics from the whole sample. As can be seen, the resulting distribution is characterized by significant excess kurtosis and slight positive skewness. The Bera - Jarque test for normality leads to a strong rejection of its null hypothesis.

[Insert table 1 and 2 approximately here]

To investigate the auto-correlation structure, we applied the Box - Ljung test to the auto-correlations up to lags 8, 12 and 16 for the raw data as well the squares and absolute values of returns. In harmony with empirical records, there is only slight auto-correlation in the returns themselves, but highly significant auto-correlation in the squares and absolute values. Since with samples of that size, we are able to detect even very small degrees of auto-correlation with high reliability, we would not expect the results of the Box-Ljung test to be insignificant (in fact, they allow rejection of the null of no auto-correlation even for the raw returns). However, what is interesting here is that the statistics are orders of magnitude larger for the squares and absolute values of returns.

The highly significant entries for the latter transformations lead to the questions of whether these time series are able to mimic the empirical observations of long-term dependence, defined as an hyperbolic decline of the auto-correlation function:

$$ACF(\tau) \approx \tau^{-\gamma}. \tag{18}$$

where γ is the decay constant. To this end we estimate the parameter of fractional differencing, denoted by **d**, from a regression in frequency space following the approach by Geweke and Porter - Hudak (GPH) [18], and also the Hurst exponent **H** from Detrended Fluctuation Analysis (DFA) [33], see tables 3 and 4.

[Insert tables 3 and 4 approximately here]

The GPH method is based on the linear regression of the log-periodogram on transformations of low frequencies of the Fourier spectrum. The estimated parameter d is related to the decay rate of the auto-correlation function by:

$$\gamma = 1 - 2d \tag{19}$$

A value of $d = 0$ would indicate absence of long memory, while d significantly above zero speaks in favor of long-term dependence. Table 3 gives summarizing results from 500 sub-samples of $2,000$ observations each, and the histograms show the distribution of the 500 estimates. As it turns out, we get results in the vicinity of zero for the raw data, but on average much higher values for the squares and absolute returns. In fact, the latter are very close to typical empirical estimates obtained with returns of various financial markets (cf. Lux and Ausloos, [28]).

Estimates from the alternative DFA methods (shown in table 4 and relative histograms) confirm these results. The relationship between the two parameters is:

$$H = 2d + 0.5 \tag{20}$$

therefore for the two methods we have a satisfactory agreement for raw returns; for absolute and squared returns, results from both methods are qualitatively similar, albeit with some divergence in the numerical values. The later might be explained, however, by different small sample biases of both estimators.

The appearance of long term dependence is particularly interesting since simple inspection of the model, in fact, indicates that **it does not exhibit this feature**: any memory in the system is wiped out by stochastic fluctuations between the two modes of the distribution and the time needed to switch from one mode to the other is depended on the past behavior of the system. However, it is well known that Markov regime-switching models can indeed 'erroneously' give the impression of long - term dependence [26, 1, 19, 11]. The mechanism here is similar to Markov switching processes, which might explain the impression of long - term memory. A similar result is found in Kirman and Teyssiére [23], who study a more complicated foreign exchange market model in which Kirman's herding model is combined with a monetary model *á la* Frankel and Froot [14].

Turn now to the unconditional distribution of the synthetic data. To complement the results for kurtosis, we compute the so-called tail index to get information about the heaviness of the tails of the simulated data. Empirical research indicates again a hyperbolic relationship for the decay of the probability in the outer part of the return distribution, following:

$$P(|R_t| > X) \approx X^{-\alpha} \tag{21}$$

with α usually in the range of $[2.5, 5]$, (cf. Lux and Ausloos, [28]). Here we applied the usual maximum likelihood estimator proposed by Hill [20], using the same 500 sub-samples and tail sizes of 10, 5 and 2.5%. Both the range of the estimators and the tendency towards slightly increasing numbers are in good harmony with empirical results (see table 5 and the pertinent histograms for more details).

[Insert table 5 approximately here]

The unit-root hypothesis of the financial data is another well-established stylized fact of asset prices [35], usually interpreted as a consequence of market efficiency. In other words, one is usually not able to reject the null hypothesis that the price of financial assets follows a random walk or martingale process. To test the unit root, we applied the famous Dickey-Fuller test to sub-samples of different lengths, (from 500 to 10000, see table 6), in order to check wether the simulated time series show the same pattern as the empirical data.

[Insert table 6 approximately here]

As can be seen from table 6, we cannot reject the null hypothesis of unit-root using a one-sided test for all the sub-samples considered. Conversely, applying a two-sided test, we observe several cases of rejections in favor of an explosive root of the dynamics. Inspection shows that these cases of rejection of a unit root in favor of explosive dynamics are driven by switching between the two modes of the distribution. The fast change in the majority of the noise traders creates the impression of an exponential increase of the price ($\rho > 1$) for particular choices of the size of the sub-samples, even tough the time series of the price is bounded. However, with longer sizes we observe fewer rejections also for a two-sided test, since the time series, then, runs over several transitions between the two "equilibria".

5 Discovering the Asymptotic Behavior

The incongruity between the theoretical properties of the model (absence of long memory) and the results of the statistical investigation, described in the previous paragraph, at the end should be a 'finite size' effect (even though one might recover the 'true' behavior only with immense amounts of data). To show the transition towards its true behavior in the case of apparent long-term dependence of volatility, it is necessary to study the asymptotic correlation properties of the time series.

[Insert figure 3 approximately here]

To this end, figure 3 shows the Hurst exponent, estimated with DFA, as a function of different time windows (ranging from 10 to $5 \cdot 10^5$ time steps) for raw, squared and absolute returns.

Concerning the raw returns for a time window of few thousands data points, we observe a vanishing Hurst exponent, that approaches zero for longer time windows. This behavior can be explained by the boundedness of the time series of the price, which leads to a constant variance of returns. But if we restrict our time horizon to few hundreds time steps, the Hurst exponent is close to 0.5, the typical value for a random walk.

The estimation for the time series of squared and absolute returns shows different properties. In the first part it has a value greater than 0.5, within the characteristic interval for long memory processes, but for $\sim 10^4$ time steps, it declines to the typical value for the random walk; and at the end we observe a convergence to zero that means indication of a bounded time series.

The explanation of these results lies in the oscillatory pattern of the price. These oscillations create a characteristic time scale T_0 (see equation (14)), inside which the time series is a random walk, with a linear increase of the variance over time. But in the case of longer time series (the size of the sample T several times greater than T_0), the variance reaches a constant value since it, then, constitutes average over numerous oscillations.

In terms of absolute returns, these oscillations create a kind of regime switching between a calm period and a turbulent one, giving the impression of a long memory process, at least for time windows not too large compared to T_0. This effect vanishes as soon as the size of the sample is long enough.

The time scale T_0, therefore, regulates the necessary amount of data for recovering the true behavior of the model. Samples of smaller size, on the other hand, give rise to different 'spurious' characteristics, which are, in fact, in good agreement with the empirical data; from (14) we can even calculate the scaling of the necessary sample size with respect to the parameters of the model.

6 Conclusions

This paper has presented an extremely simple variant of a noise trader/infection model. In contrast to many other contributions in the literature on artificial financial markets, it belongs to a class of models whose dynamical behavior is well understood. In particular, we know that as a bounded Markovian process with a bistable limiting distribution, the model should lack any 'true' scaling properties. Nevertheless, applying the usual statistical tests to simulated data, we find 'apparent' scaling with quite close agreement with empirically observed exponents. This shows that the difficulty to distinguish between true and spurious scaling is not confined to particular stochastic processes, but may also emerge in the area of multi-agent behavioral models. We argue that such apparent scaling might also occur in other models presented in the literature.

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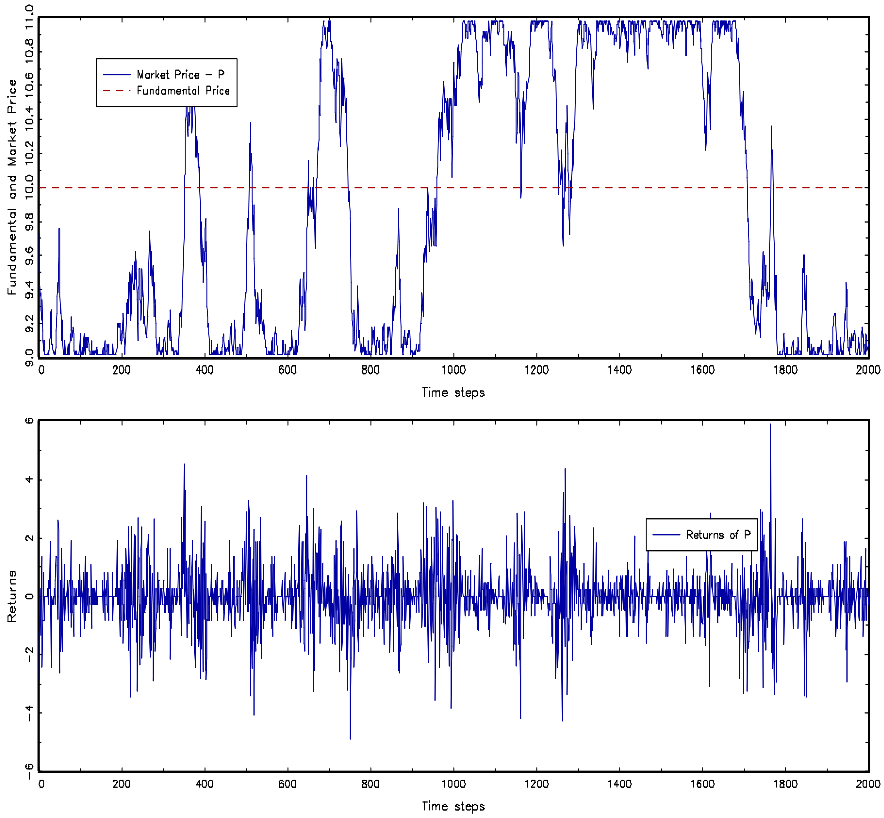


Figure 1: *The upper panel shows the behavior of the fundamental (simply assumed to be constant) and the market price from a typical simulation. The lower panel shows the returns of the market price, computed as log increments over unit time interval. Underlying parameters of this run:* $N = 100$, $\nu = 1$, $p_F = 10$.

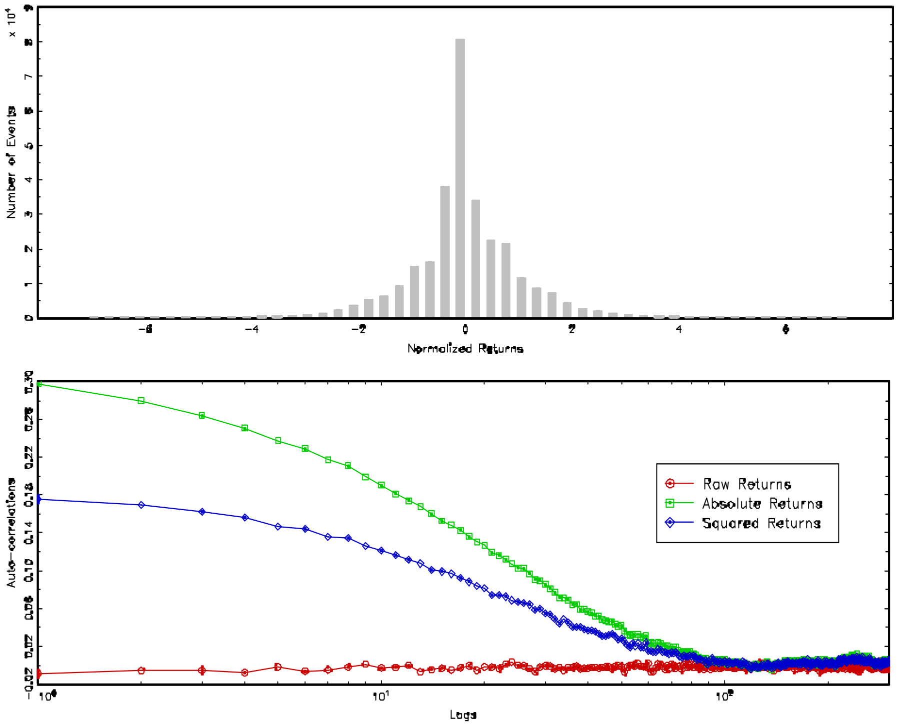


Figure 2: *The upper panel shows the distribution of normalized returns; notice the leptokurtic shape. The lower panel shows the auto-correlation function of raw, squared and absolute returns. Parameters:* $N = 100$, $\nu = 1$, $p_F = 10$, *number of events* $3 \cdot 10^5$.

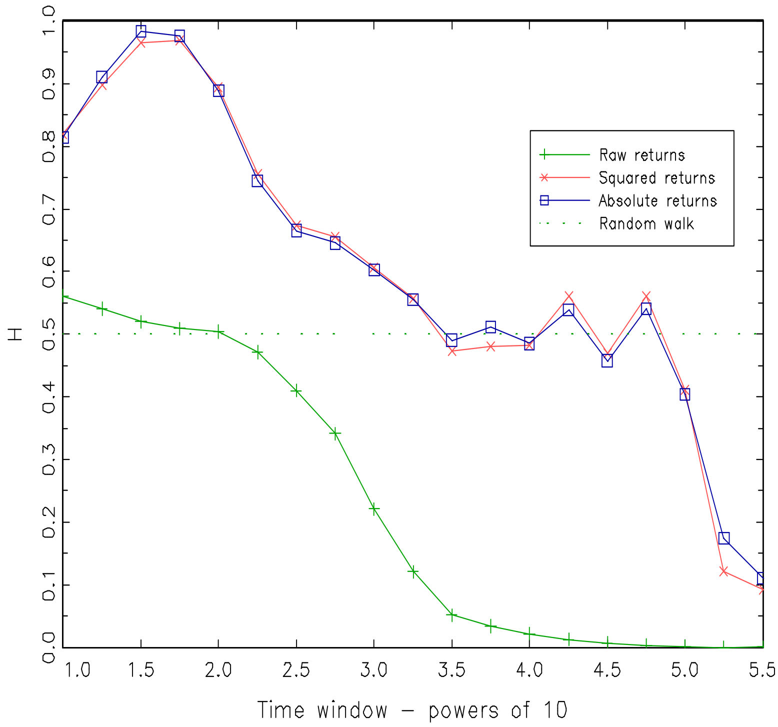


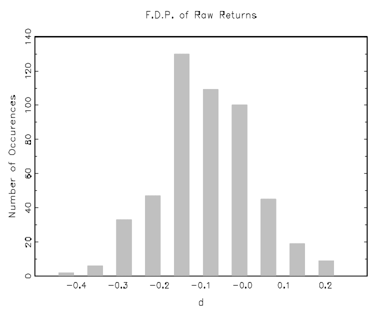
Figure 3: *Estimated Hurst exponent for raw, squared and absolute returns as a function of different time windows calculated via detrented fluctuation analysis. The dashed line is the typical value for a random walk.* $T_0 = 461$ *for* $N = 100$*, which corresponds to* $10^{2.65}$.

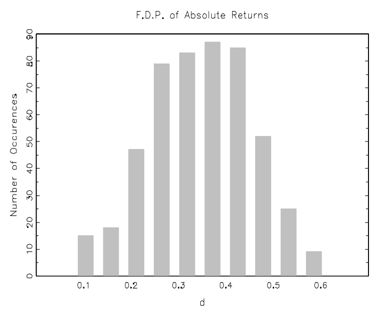
Mean	$4.14 \cdot 10^{-7}$
Variance	$7.52 \cdot 10^{-5}$
Kurtosis	2.67
Skewness	0.057
Bera-Jarque test	59,571
(*Probability*)	(*0.000*)

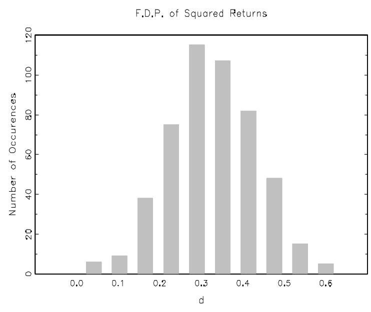
Table 1: *Sample statistics of returns.*

	8	12	16
R_t	58.59	96.17	116.74
R_t^2	40,198	52,270	61,611
$\lvert R_t \rvert$	107,132	138,035	160,551

Table 2: *Results of Box-Ljung test.*

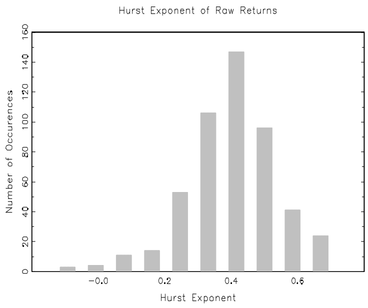


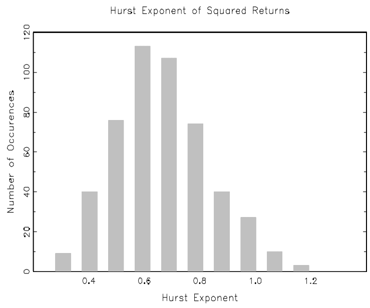


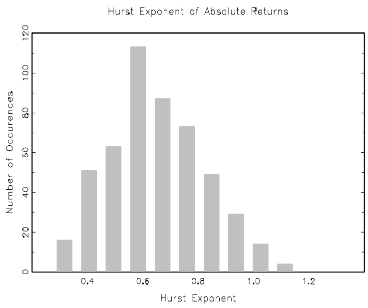


	Mean	Minimum	Maximum
R_t	-0.09	-0.46	0.24
R_t^2	0.33	0.01	0.63
$\|R_t\|$	0.35	0.08	0.61

Table 3: *Estimated parameters of fractional differencing for* 500 *sub-samples.*

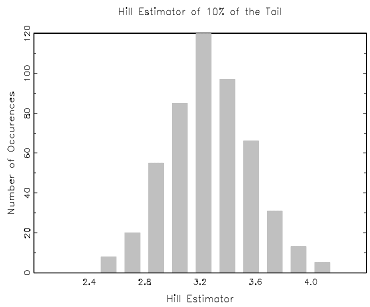


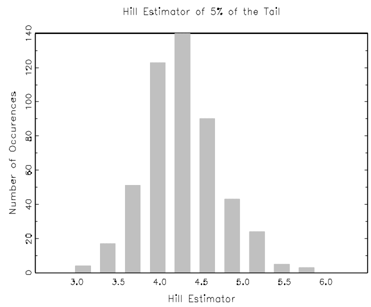


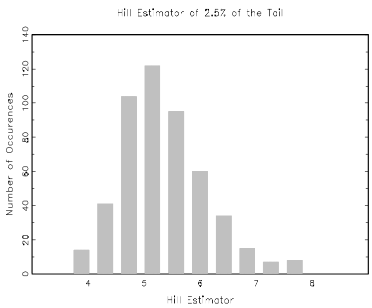


	Mean	Minimum	Maximum
R_t	0.40	−0.13	0.71
R_t^2	0.66	0.26	1.21
$\|R_t\|$	0.65	0.27	1.15

Table 4: *Hurst Exponent from DFA for* 500 *sub-samples.*







	Mean	Minimum	Maximum
10%	3.27	2.45	4.17
5%	4.27	2.92	5.92
2.5%	5.33	3.66	7.90

Table 5: *Tail index estimators for* 500 *sub-samples.*

Size of the sub-sample	Range of ρ	One-sided test [a]	Two-sided test [a]
500	$0.99998279 - 1.00000171$	0(2000)	615(2000)
2000	$0.99999562 - 1.00000042$	0(500)	114(500)
5000	$0.99999824 - 1.00000016$	0(200)	28(200)
10000	$0.99999909 - 1.00000008$	0(100)	0(100)

Table 6: *Results of Unit-Root test. (a) Number of rejections at* 95% *level.*