Appendix A: The Moments of a Causal Multi-Fractal Process with A Lognormal Volatility Cascade

1. Moments of Log Increments of the Volatility Dynamics

To derive the auto-covariances of the log differences of the innovations of the compound process, we first consider the log differences of the directing volatility process itself, i.e. $\eta_{t,1} = \ln(\mu_t) - \ln(\mu_{t-1})$ with:

$$\text{(A1)}\quad \ln\mu_t = \ln(2^k \prod_{i=1}^{k} m_t^{(i)}) = \sum_{i=1}^{k} \ln(m_t^{(i)}) + k\cdot\ln(2) = \sum_{i=1}^{k} \varepsilon_t^{(i)} + k\cdot\ln(2)\,.$$

Note, that obviously, $E[\eta_{t,1}] = E\left\{\sum_{i=1}^{k}\varepsilon_t^{(i)} - \sum_{i=1}^{k}\varepsilon_{t-1}^{(i)}\right\} = 0$. Now turn to auto-covariances:

$$\text{(A2)}\quad E[\eta_{t+1,1}\eta_{t,1}] = E\left\{\left[\sum_{i=1}^{k}(\varepsilon_{t+1}^{(i)} - \varepsilon_t^{(i)})\right]\cdot\left[\sum_{i=1}^{k}(\varepsilon_t^{(i)} - \varepsilon_{t-1}^{(i)})\right]\right\}.$$

Computation of this expression is greatly facilitated when recognizing that of its k^2 individual components of the form $(\varepsilon_{t+1}^{(i)} - \varepsilon_t^{(i)})(\varepsilon_t^{(j)} - \varepsilon_{t-1}^{(j)})$ only those with $i = j$ make a non-zero contribution. The expectation of all the remaining components with $i \neq j$ is identical equal to zero simply because of independence of draws at different stages of the volatility cascade. It, then, remains to consider the remaining k cases with $i = j$, $i = 1,2,\ldots,k$. For $k = 1$, we find that:

$$\text{(A3)}\quad E[(\varepsilon_{t+1}^{(1)} - \varepsilon_t^{(1)})(\varepsilon_t^{(1)} - \varepsilon_{t-1}^{(1)})] = E[\varepsilon_{t+1}^{(1)}\varepsilon_t^{(1)}] - E[(\varepsilon_t^{(1)})^2] - E[\varepsilon_{t+1}^{(1)}\varepsilon_{t-1}^{(1)}] + E[\varepsilon_t^{(1)}\varepsilon_{t-1}^{(1)}]$$

makes a non-zero contribution only if the first multiplier changes twice from t-1 to t+1. Since the probability of this event is $(\frac{1}{2^{k-1}})^2$, one obtains:

$$\text{(A4)}\quad \begin{aligned} &E[(\varepsilon_{t+1}^{(1)} - \varepsilon_t^{(1)})(\varepsilon_t^{(1)} - \varepsilon_{t-1}^{(1)})] = \\ &= (\frac{1}{2^{k-1}})^2\left\{E[\varepsilon_{t+1}^{(1)}]^2 - E[(\varepsilon_t^{(1)})^2\right\} = (\frac{1}{2^{k-1}})^2(\lambda^2 - \lambda^2 - \sigma_\varepsilon^2) = (\frac{1}{2^{k-1}})^2(-\sigma_\varepsilon^2)\end{aligned}$$

Similarly, non-zero contributions for $i = 2,3,\ldots k$, are obtained only if the pertinent multiplier changes twice. This happens automatically, if a higher multiplier has been renewed two times (since by the construction of the process this wipes out the memory at all lower levels of the cascade). If not, there is still a chance for one or two new draws between t-1 and t+1 to occur at the level i itself. Adding up the relevant probabilities, we find for i = 2:

$$\text{(A5)}\quad \begin{aligned} &E[(\varepsilon_{t+1}^{(2)}-\varepsilon_t^{(2)})(\varepsilon_t^{(2)}-\varepsilon_{t-1}^{(2)})] = \\ &= \left[(\frac{1}{2^{k-1}})^2 + 2\cdot(1-\frac{1}{2^{k-1}})\frac{1}{2^{k-1}}\frac{1}{2^{k-2}} + (1-\frac{1}{2^{k-1}})^2(\frac{1}{2^{k-1}})^2\right]\cdot(-\sigma_\varepsilon^2) \end{aligned}$$

Increasingly complex formulas are obtained when considering i = 3,4,…, k. For i > 1, we can express the pertinent contributions compactly as:

$$\text{(A6)}\quad E[(\varepsilon_{t+1}^{(i)}-\varepsilon_t^{(i)})(\varepsilon_t^{(i)}-\varepsilon_{t-1}^{(i)})] =$$

$$= \left\{\left[\prod_{j=1}^{i-1}{}^{+}(1-\frac{1}{2^{k-j}})^2\right](\frac{1}{2^{k-i}})^2 + \sum_{j=1}^{i-1}\left[\prod_{n=1}^{j-1}{}^{+}(1-\frac{1}{2^{k-n}})^2\right]2(1-\frac{1}{2^{k-j}})\frac{1}{2^{k-j}}\left[\prod_{m=j+1}^{i-1}{}^{+}(1-\frac{1}{2^{k-m}})\right]\frac{1}{2^{k-i}}\right\}\cdot(-\sigma_\varepsilon^2)$$

with the notational convention $\prod_a^b{}^{+}() = \prod_a^b()$ if $b \geq a$ and 1 if $b < a$. In (A6), the first entry on the right-hand side covers all cases with two replacements occurring at the same level of the cascade, while the second entry captures cases with replacements taking place at two different levels: the first at some j < i and the second at level i.

Summing over the contributions for all multipliers i = 1,…, k, we arrive at the closed form solution for the autocovariance at lag 1:

$$\text{(A7)}\quad \begin{aligned} &E(\eta_{t+1,1}\cdot\eta_{t,1}) = -\sum_{j=1}^{k}(k-j+1)\left[\prod_{i=1}^{j-1}{}^{+}(1-\frac{1}{2^{k-i}})^2\right](\frac{1}{2^{k-j}})^2\sigma_\varepsilon^2 - \\ &\sum_{j=2}^{k}(k-j+1)\left(\sum_{i=1}^{j-1}\left[\prod_{n=1}^{i-1}{}^{+}(1-\frac{1}{2^{k-n}})^2\right]2(1-\frac{1}{2^{k-i}})\frac{1}{2^{k-i}}\left[\prod_{m=i+1}^{j-1}{}^{+}(1-\frac{1}{2^{k-m}})\right]\frac{1}{2^{k-j}}\right)\cdot\sigma_\varepsilon^2 \end{aligned}$$

Note that autocovariances at higher lags $\tau > 1$,

$$\text{(A8)}\quad E[\eta_{t+\tau,1}\cdot\eta_{t,1}] = E\left\{\left[\sum_{i=1}^{k}(\varepsilon_{t+\tau}^{(i)}-\varepsilon_{t+\tau-1}^{(i)})\right]\cdot\left[\sum_{i=1}^{k}(\varepsilon_{t}^{(i)}-\varepsilon_{t-1}^{(i)})\right]\right\},$$

are all equal to zero because of the independence of changes between t-1 and t, and between t+τ-1 and t+ τ (τ > 1), respectively. This underscores the *short*-memory property of the process governing log increments.

One might, however, also be interested in the autocovariance of log increments recorded over time intervals longer than one period, e.g.:

$$\text{(A9)}\quad E[\eta_{t+T,T}\cdot\eta_{t,T}] = E\left\{\left[\sum_{i=1}^{k}(\varepsilon_{t+T}^{(i)}-\varepsilon_{t}^{(i)})\right]\cdot\left[\sum_{i=1}^{k}(\varepsilon_{t}^{(i)}-\varepsilon_{t-T}^{(i)})\right]\right\}.$$

It is straightforward to derive the closed-form solution for these moments from an appropriate adaptation of the reasoning underlying eq. (A7). Again, non-zero contributions are only obtained from products of multipliers of the same stage i. Furthermore, two changes of this multiplier must have occurred between t-T and t, and t and t+T, respectively. Plugging in the pertinent probabilities for the later events, the following generalization of (A7) is obtained:

(A10)

$$E(\eta_{t+T,T}\cdot\eta_{t,T}) = -\sum_{j=1}^{k}(k-j+1)\left[\prod_{i=1}^{j-1}{}^{+}(1-\frac{1}{2^{k-i}})^{2T}\right]\left(1-(1-\frac{1}{2^{k-j}})^{T}\right)^{2}\sigma_{\varepsilon}^{2} -$$

$$\sum_{j=2}^{k}(k-j+1)\left(\sum_{i=1}^{j-1}\left[\prod_{n=1}^{i-1}{}^{+}(1-\frac{1}{2^{k-n}})^{2T}\right]2(1-\frac{1}{2^{k-i}})^{T}\left(1-(1-\frac{1}{2^{k-i}})^{T}\right)\left[\prod_{m=i+1}^{j-1}{}^{+}(1-\frac{1}{2^{k-m}})^{T}\right]\left(1-(1-\frac{1}{2^{k-j}})^{T}\right)\right)\cdot\sigma_{\varepsilon}^{2}$$

Again, covariances of T-step log increments over more than T lags are all equal to zero because of independence of the two entries in the product:

$$\text{(A11)}\quad E[\eta_{t+T+\tau,T}\cdot\eta_{t,T}] = E\left\{\left[\sum_{i=1}^{k}(\varepsilon_{t+T+\tau}^{(i)}-\varepsilon_{t+\tau}^{(i)})\right]\cdot\left[\sum_{i=1}^{k}(\varepsilon_{t}^{(i)}-\varepsilon_{t-T}^{(i)})\right]\right\}=0 \quad ,\ \forall\tau\geq 1.$$

The second set of moment conditions considered in the GMM estimator proposed in the main text is *autocovariances of squared entries of log increments*, e.g.

$$\text{(A12)}\quad E[\eta_{t+1,1}^{2}\cdot\eta_{t,1}^{2}] = E\left\{\left(\sum_{i=1}^{k}(\varepsilon_{t+1}^{(i)}-\varepsilon_{t}^{(i)})\right)^{2}\cdot\left(\sum_{i=1}^{k}(\varepsilon_{t}^{(i)}-\varepsilon_{t-1}^{(i)})\right)^{2}\right\}.$$

Here things become somewhat more involved. In order to motivate the resulting solution, let us distinguish between three different cases of individual summands on the RHS of (A12) with potential non-zero contributions:

(i) $E\left[\left(\varepsilon_{t+1}^{(i)}-\varepsilon_t^{(i)}\right)^2\left(\varepsilon_t^{(i)}-\varepsilon_{t-1}^{(i)}\right)^2\right]=-6\sigma_\varepsilon^4$ if $\varepsilon_{t+1}^{(i)}\neq\varepsilon_t^{(i)}\neq\varepsilon_{t-1}^{(i)}$ and 0 otherwise.

As we already know, the sum over the probabilities for all i for this condition to be met is:

$$\chi_1=\sum_{j=1}^{k}(k-j+1)\left[\prod_{i=1}^{j-1}{}^{+}(1-\frac{1}{2^{k-i}})^2\right](\frac{1}{2^{k-j}})^2$$
$$+\sum_{j=2}^{k}(k-j+1)\left(\sum_{i=1}^{j-1}\left[\prod_{n=1}^{i-1}{}^{+}(1-\frac{1}{2^{k-n}})^2\right]2(1-\frac{1}{2^{k-i}})\frac{1}{2^{k-i}}\left[\prod_{m=i+1}^{j-1}{}^{+}(1-\frac{1}{2^{k-m}})\right]\frac{1}{2^{k-j}}\right)$$

(ii) for $i\neq j$: $E\left[\left(\varepsilon_{t+1}^{(j)}-\varepsilon_t^{(j)}\right)^2\left(\varepsilon_t^{(i)}-\varepsilon_{t-1}^{(i)}\right)^2\right]=-4\sigma_\varepsilon^4$ if both $\varepsilon_{t+1}^{(j)}\neq\varepsilon_t^{(j)}$ and $\varepsilon_t^{(i)}\neq\varepsilon_{t-1}^{(i)}$ hold, and 0 otherwise.

The sum of the relevant probabilities is:

$$\chi_2=\sum_{i=1}^{k}(k-i)(k-i+1)\left[\prod_{h=1}^{i-1}{}^{+}(1-\frac{1}{2^{k-h}})^2\right](\frac{1}{2^{k-i}})^2$$
$$+\sum_{j=2}^{k}\sum_{i=1}^{j-1}\left\{\left(\prod_{n=1}^{i-1}{}^{+}\left(1-\frac{1}{2^{k-n}}\right)^2\right)\cdot\frac{1}{2^{k-i}}\left(1-\frac{1}{2^{k-i}}\right)\left(\prod_{m=i+1}^{j-1}{}^{+}\left(1-\frac{1}{2^{k-m}}\right)\right)\frac{1}{2^{k-j}}\right\}$$

The first summand stems from those cases where changes in one volatility component i occur both at t and t+1 and all higher level components remain constant. Due to our hierarchical construction this also leads to all lower level components j > i to be renewed twice. One can easily check that there are exactly ∑(k-i)(k-i+1) such cases. The second part covers those cases in which component i changes between t-1 and t and j changes 'autonomously' in the interval from t to t+1.

(ii) for $i\neq j$: $E[(\varepsilon_{t+1}^{(j)}-\varepsilon_t^{(j)})(\varepsilon_t^{(j)}-\varepsilon_{t-1}^{(j)})(\varepsilon_{t+1}^{(i)}-\varepsilon_t^{(i)})(\varepsilon_t^{(i)}-\varepsilon_{t-1}^{(i)})]=-\sigma_\varepsilon^4$ if both $\varepsilon_{t+1}^{(i)}\neq\varepsilon_t^{(i)}\neq\varepsilon_{t-1}^{(i)}$ and $\varepsilon_{t+1}^{(j)}\neq\varepsilon_t^{(j)}\neq\varepsilon_{t-1}^{(j)}$ hold, and 0 otherwise.

The non-zero case occurs with accumulated probabilities:

$$\chi_3 = \sum_{i=1}^{k} 2(k-i)(k-i+1)\left[\prod_{h=1}^{i-1}{}^{+}(1-\frac{1}{2^{k-h}})^2\right](\frac{1}{2^{k-i}})^2$$
$$+\sum_{i=2}^{k} 4(k-i)(k-i+1)\sum_{h=1}^{i-1}\left\{\left(\prod_{n=1}^{h-1}{}^{+}\left(1-\frac{1}{2^{k-n}}\right)^2\right)\cdot\frac{1}{2^{k-h}}\left(1-\frac{1}{2^{k-n}}\right)\left(\prod_{m=h+1}^{i-1}{}^{+}\left(1-\frac{1}{2^{k-m}}\right)\right)\frac{1}{2^{k-i}}\right\}$$

Again, the accumulated probabilities consist of one entry covering those cases in which two changes occur at some $i < j$ so that non-constant multipliers at stage i (or at some earlier stage) also imply that j undergoes two changes between t-1 and t+1. The factor 2(k-i)(k-i+1) gives the number of possible combinations (i,j) that belong to this category. Also, the second entry stands for the autonomous changes at level j multiplied by the number of pertinent permutations.

Overall, we, therefore arrive at the following formula for lag 1 autocovariances of the squares of log differences of the multi-fractal volatility process:

(A13) $E[\eta^2_{t+1,1}\cdot\eta^2_{t,1}] = -(6\chi_1 + 4\chi_2 + \chi_3)\sigma^4_\varepsilon$.

It is straightforward to modify (A13) appropriately in order to arrive at similar formulas for autocovariances of arbitrary T step increments $E[\eta^2_{t+T,T}\cdot\eta^2_{t,T}]$ and it is similarly straightforward to show that autocovariances over more than one lag, $E[\eta^2_{t+T+\tau,T}\cdot\eta^2_{t,T}]$ are identically equal to zero for all T and $\tau \geq 1$.

2. Moments of Log Increments of the Compound Process

Applying the above insights to the compound model, we have to take into account the additional contribution of the normally distributed innovations u_t.

From (18) in the main text, we get the differences of the log innovations, $\xi_t = \ln|x_t| - \ln|x_{t-1}|$, of the compound process:

(A14) $\xi_t = \ln|x_t| - \ln|x_{t-1}| = 0.5\sum_{i=1}^{k}(\varepsilon_t^{(i)} - \varepsilon_{t-1}^{(i)}) + \ln|u_t| - \ln|u_{t-1}|$.

Clearly, $E[\xi_t] = 0$.

Again, we are interested in the covariances of both the log increments and their squared entries. For the former, we obtain, for arbitrary T,

$$\begin{aligned}
&E[\xi_{t+T,T}\cdot\xi_{t,T}] \\
\text{(A15)}\qquad &= E\left\{\left(0.5\sum_{i=1}^{k}(\varepsilon_{t+T}^{(i)}-\varepsilon_{t}^{(i)})+\ln|u_{t+T}|-\ln|u_t|\right)\cdot\left(0.5\sum_{i=1}^{k}(\varepsilon_{t}^{(i)}-\varepsilon_{t-T}^{(i)})+\ln|u_t|-\ln|u_{t-T}|\right)\right\} \\
&= 0.25\cdot E[\eta_{t+T,T}\cdot\eta_{t,T}]+\left[E\ln|u_t|\right]^2-E\left[(\ln|u_t|)^2\right]
\end{aligned}$$

in which all the cross-products between η_t and u_t terms vanish because of the assumed independence between both processes.

Slightly more effort has to be put into determination of the autocovariances of the squared entries which are found to yield:

$$\begin{aligned}
&E[\xi^2_{t+T,T}\cdot\xi^2_{t,T}] \\
&= E\left\{\left(0.5\sum_{i=1}^{k}(\varepsilon_{t+T}^{(i)}-\varepsilon_{t}^{(i)})+\ln|u_{t+T}|-\ln|u_t|\right)^2\cdot\left(0.5\sum_{i=1}^{k}(\varepsilon_{t}^{(i)}-\varepsilon_{t-T}^{(i)})+\ln|u_t|-\ln|u_{t-T}|\right)^2\right\} \\
\text{(A16)}\qquad &= 0.25^2\cdot E[\eta^2_{t+T,T}\cdot\eta^2_{t,T}]-\left\{E[\Delta\eta^2_{t,T}]-E[\eta_{t+T,T}\cdot\eta_{t,T}]\right\}\cdot\left\{\left[E\ln|u_t|\right]^2-E\left[(\ln|u_t|)^2\right]\right\} \\
&+3E\left[(\ln|u_t|)^2\right]^2-4E\left[\ln|u_t|\right]\cdot E\left[(\ln|u_t|)^3\right]-E\left[(\ln|u_t|)^4\right]
\end{aligned}$$

Note that the log moments of the Normal distribution can be calculated using the Gamma function and its derivatives. For the first log moment we get:

$$\text{(A17)}\quad \int_0^\infty(\ln u)e^{-au^2}du=\int_0^\infty 0.25(\ln y-\ln a)e^{-y}(ay)^{-0.5}dy=\frac{1}{4\sqrt{a}}\Gamma'(0.5)-\frac{1}{4\sqrt{a}}(\ln a)\Gamma(0.5)$$

In order to proceed from the first integral to the second, one uses the substitution $y = ax^2$ ($a=-(2\sigma_u^2)^{-1}$). For the second to fourth moment one obtains via similar operations:

$$\text{(A18)}\quad \int_0^\infty(\ln u)^2e^{-au^2}du=\frac{1}{8\sqrt{a}}\Gamma''(0.5)-\frac{1}{4\sqrt{a}}(\ln a)\Gamma'(0.5)+\frac{1}{8\sqrt{a}}(\ln a)^2\Gamma(0.5)$$

$$\begin{aligned}
&\int_0^\infty(\ln u)^3e^{-au^2}du= \\
\text{(A19)}\quad &\frac{1}{16\sqrt{a}}\Gamma'''(0.5)-\frac{3}{16\sqrt{a}}(\ln a)\Gamma''(0.5)+\frac{3}{16\sqrt{a}}(\ln a)^2\Gamma'(0.5)-\frac{1}{16\sqrt{a}}(\ln a)^3\Gamma(0.5)
\end{aligned}$$

(A20) $\int_0^\infty (\ln u)^4 e^{-au^2} du =$

$$\frac{1}{32\sqrt{a}}\Gamma^{(iv)}(0.5) - \frac{4}{32\sqrt{a}}(\ln a)\Gamma'''(0.5) + \frac{6}{32\sqrt{a}}(\ln a)^2\Gamma''(0.5) - \frac{4}{32\sqrt{a}}(\ln a)^3\Gamma'(0.5) + \frac{1}{32\sqrt{a}}(\ln a)^4\Gamma(0.5)$$

3. Moments of the Compound Process

For computing the linear forecasts of future volatility, we also need to derive the autocovariances of the original multi-fractal data before log differencing, i.e. $\text{cov}[x_{t+T}^2 x_t^2]$.

In order to do so, let us first note that:

(A21) $E[x_{t+T}^2 x_t^2] = E[\mu_{t+T}\mu_t] \cdot E[u_t^2]^2$.

Starting with the first lag, we can derive again a simple formula by considering all possible scenarios of replacements of multipliers. First, when the multiplier of highest order, $m_t^{(1)}$, is replaced by a new draw at t+1, μ_t and μ_{t+1} would have no joint elements and, hence, the expectation of their product would be $E[m_t^{(i)}]^{2k}$. This happens with probability $2^{-(k-1)}$. If this event does not occur, we get replacement of the second multiplier with probability $(1-2^{-(k-1)})\, 2^{-(k-2)}$ which leads to an expected value of the product $\mu_t\, \mu_{t+1}$ equal to $E[m_t^{(i)}]^{2k-2}E[(m_t^{(i)})^2]$ since μ_t and μ_{t+1} have one joint and k-1 different multipliers. Proceeding in this manner, we arrive at:

(A22) $$E[\mu_{t+1}\mu_t] = \sum_{n=1}^{k-1}\left(\prod_{j=1}^{n}(1-\frac{1}{2^{k-j}})\right)\cdot\frac{1}{2^{k-n-1}}E\left[\left(m_t^{(i)}\right)^2\right]^n E\left[m_t^{(i)}\right]^{2k-2n} + \frac{1}{2^{k-1}}E\left[m_t^{(i)}\right]^{2k}.$$

One only needs to replace the probabilities for constancy and replacement of the multipliers in one step by the probabilities of constancy over T steps or *at least one* replacement within T steps, respectively, to arrive at the more general expression:

(A23) $$\begin{aligned} E[\mu_{t+T}\mu_t] = &\sum_{n=1}^{k-1}\left(\prod_{j=1}^{n}(1-\frac{1}{2^{k-j}})^T\right)\cdot\left(1-(1-\frac{1}{2^{k-n-1}})^T\right)E\left[\left(m_t^{(i)}\right)^2\right]^n E\left[m_t^{(i)}\right]^{2k-2n} \\ &+\left(1-(1-\frac{1}{2^{k-1}})^T\right)E\left[m_t^{(i)}\right]^{2k}. \end{aligned}$$

(A21) is easily implemented by noting that: $E[u_t^2]^2 = \sigma_u^4$.

Finally, we note that from (9) in the main text,

$$\text{(A24)} \quad E[x_t^4] = 2^{2k} \cdot \left(2^{-k}\right)^{2\lambda - 4(\lambda - 1)} \cdot 3\sigma_u^4$$

so that we have all information needed to implement the linear predictors derived from the multi-fractal cascade.

Appendix B: GARCH and FIGARCH Parameter Estimates

		μ	ρ	ω	β	α	φ	d	Logl	AIC	BIC
NYCI	GARCH	0.050 (0.010)	0.114 (0.016)	0.013 (0.003)	0.913 (0.010)	0.072 (0.007)			-5225.21	10460.43	10492.54
	FIGARCH	0.051 (0.010)	0.112 (0.016)	0.024 (0.012)	0.6636 (0.0972)		0.442 (0.100)	0.350 (0.106)	**-5199.82**	**10411.65**	**10450.19**
DAX	GARCH	0.042 (0.013)	0.077 (0.017)	0.044 (0.007)	0.843 (0.015)	0.125 (0.013)			-6232.18	12474.36	12506.41
	FIGARCH	0.045 (0.014)	0.074 (0.019)	0.075 (0.030)	0.3440 (0.0802)		0.052 (0.044)	0.378 (0.119)	**-6200.93**	**12413.86**	**12452.32**
US$-DEM	GARCH	-0.002 (0.010)	-0.042 (0.016)	0.018 (0.003)	0.876 (0.011)	0.095 (0.009)			**-4673.87**	**9357.74**	**9389.68**
	FIGARCH	-0.003 (0.011)	-0.040 (0.024)	0.022 (0.011)	0.601 (0.127)		0.215 (0.063)	0.467 (0.161)	-4676.51	9365.02	9403.35
GOLD	GARCH	-0.006 (0.014)	-0.077 (0.017)	0.024 (0.003)	0.899 (0.009)	0.092 (0.009)			-6648.87	13307.74	13339.71
	FIGARCH1	0.004 (0.004)	-0.095 (0.021)	0.014 (0.006)	0.925 (0.019)		0.124 (0.064)	0.999 (.)	**-6637.14**	**13286.28**	**13324.65**
	FIGARCH2	0.004 (0.023)	-0.095 (0.022)	0.067 (0.075)	0.600 (0.397)		0.382 (0.260)	0.408 (0.241)	-6639.15	13290.30	13328.67

Note: FIGARCH estimates are based on a truncation lag T = 1000 together with 1000 presample values set equal to the variance of the time series. Logl is the maximized log-likelihood and AIC and BIC are Akaike and Bayesian information criteria.